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Date 3/3/91
MULTISENSOR FUSION OF CONTACT DATA

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee, Knoxville

Joel W. Spears
May 1991
ACKNOWLEDGEMENTS

I appreciate everyone who has helped in any way with this research. I wish to mention in particular the members of my Committee, Dr. Dragana Brzakovich, Dr. W. L. Green, and Dr. M. A. Abidi, my Committee Chairman and Faculty Advisor.

I am also grateful to Perceptrics Corporation and Texas Instruments for sponsoring this research. NASA also sponsored research during which I became familiar with the Robotic Workstation and the Computer Vision and Robotics Laboratory.

Janet Smith provided valuable assistance in the preparation of this document. I wish to also thank my colleagues and friends at the Computer Vision and Robotics Laboratory for their helpful assistance.
ABSTRACT

In this thesis, evidence showing the need for robotics is first presented. Then, sensing is shown to be a necessary component of practical robotic systems. Our focus is then narrowed to contact sensing, which is shown to be useful and required in many applications. The need for multiple sensors and for sensor fusion is next presented. Fusion is defined and illustrated by an example. Next, the subject of contact sensing is taken up in some detail. Various methods of detecting contact, force, slip, torque, etc. are described. Criteria for the design of contact sensors are presented. Various strategies which may be used to derive useful information for various intended applications from the use of contact sensors are examined, as well as methods of processing this data and extracting information about object shape, location, etc. Four general classes of data fusion methods are briefly described, and some recent work in sensor fusion is examined. This leads us to consider the fusion process in more detail. It is then seen that fusion methods serve to accomplish either one of two seemingly contradictory purposes: knowledge addition or knowledge verification. (Other desired characteristics of fusion functions are also given.) Various existing fusion functions are then examined. Then, utilizing the two basic principles, we derive a new analytic method uniting both principles in such a way as to allow any desired “mix” or ratio of them. In order to test these fusion methods, real force data are acquired by means of active robotic contact sensing. Features
(edges) are extracted, and the different methods applied to the fusion of the data. The relative amounts of the two principles involved in the various methods are computed using the concepts of ideal volume and actual volume and illustrated in graphs and tables. Comparison of these methods show advantages of the analytic method.
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CHAPTER 1

Introduction—Need for Robotics, Sensing, Contact

Sensing, Multi-sensing, and Sensor Fusion

1.1 Need for Robotics

One of the major goals of industry today is to increase productivity. To facilitate the achievement of this goal, special-purpose machines have been developed to speed-up manufacturing processes. However, these machines are usually expensive and inflexible. This makes the change from the manufacture of one product to another, or the change from one model to another, an expensive and difficult task. The time and expense of modifying existing manufacturing equipment and/or installing new equipment can easily and quickly become prohibitive. A robot, however, normally has the flexibility to overcome most of these problems and today is being used in Flexible Manufacturing Systems (FMS). After the initial expense of acquisition and installation of a robotic system, any required changes in the manufacturing process can usually be accomplished by reprogramming the robot(s) and/or changing end-effectors, thus reducing expenses [1]. The use of multiple end-effectors is expensive, but not as expensive as standard tooling. Expensive industrial robots typically return annually over
50% on the initial investment. Even greater returns are possible by means of the integration of different automatic production systems, such as CAD/CAM, robot carts to transport materials, robot assembly systems, and computer-based management information systems. The productivity improvement of each subsystem would be synergistically increased by that of the others. The cost of industrial robots is predicted to drop in the future, much as the cost of computing equipment and calculators has dropped since their initial appearance on the marketplace. Albus [2,3] argues that the use of robotics to increase production could increase the real wealth of an economy, making more money available for space exploration, medical research, and other worthwhile endeavors.

Potential applications of robots are not limited to industrial manufacturing. Underwater tasks, such as salvage operations, are possible applications. A robot needs no food, water, air, sleep, or entertainment and can endure much greater temperature and pressure extremes than humans. Therefore, money could be saved by not providing these things for robots. In nuclear power plants, in case of an accident, a robot could go into areas contaminated by radiation deadly to humans, and perform emergency procedures to protect life and property. There are many other potential applications in which robots may prove useful [4].
1.2 Need for Sensing

Robots on an assembly line can perform their functions tirelessly, 24 hours a day, without suffering a loss in quality of performance due to tiredness, boredom, or the monotony of their work. Yet they can fail to perform their tasks, and fail extravagantly. For example, consider a robot which is making spot welds on parts carried past the robot on an assembly line. If the motor moving the assembly line were to speed up or to be shut down and restarted so that the line becomes out of synchronization with the motions of the robot, parts might pass by the robot a few seconds too early or too late. The robot might remain idle while the assembly line paused with the part in front of the robot, and then perform the welding operation after the part had moved away and before another part had been positioned. (Or, the weld might be made on the part in the wrong place.) Both assembly line and robot are here operating with no apparent mechanical difficulties, yet the assigned task is not being performed. Another possible scenario is the failure of a motor on the robot. This could result in damage to products on the assembly line. The inadequate performance of the welding task by the robot, or the damage caused by failure of a motor on the robot, must be detected before corrective measures can be taken. The use of sensors can allow rapid detection of both these conditions.

Most robots up to the present time have been simply programmed to repeat a certain sequence of motions, which requires that objects being manipulated
or operated on must be in a certain position and orientation with respect to the robot. In an assembly line system, this in turn requires a constant line speed and synchronization with the robot’s motions, as well as exact and constant distances between objects. Small, even indetectable errors in inter-object spacing may accumulate and result in task failure and costly down-time. The correct position, orientation, and spacing of objects could be ensured by the use of special-purpose machinery, but this would be moving away from the desired flexibility of a robotic system. Also, such measures may easily prove difficult, expensive, or even impossible.

Instead of adjusting the position and orientation of the objects to that of the robot, a more feasible and appropriate solution is to adjust the robot’s position and orientation to match those of the target objects. Adjusting the robot’s position requires first that the robot be able to sense any deviations in expected object position and orientation, and secondly, that the robot be able to use this information to make appropriate corrective actions. The primary concern of this thesis will be with the first requirement.

1.3 Need for Contact Sensing

Sensing can be divided into two broad areas: (1) noncontact sensing, including vision, sonar, ultrasonic range sensing, etc., and (2) contact sensing, including the sensing of slip, torque, force, pressure, and mere contact itself, or
touch. In this thesis, the term “contact sensing” will refer to only those sensing modalities listed above.

In the initial stages of a robot’s task, when it may be necessary to locate and approach the target object before contact can be made, noncontact sensors would obviously be appropriate. However, if the object is to be picked up and manipulated (a common robotic task), noncontact sensors may not prove adequate. For example, vision sensors may be occluded at close range by the robot manipulator or end-effector itself. Also, the effect of the difference in the camera’s position and the end-effector’s position becomes more pronounced at close range. Although capacitance gauges do give accurate readings, many range sensors, such as ultrasonic ones, do not give accurate readings at short distances. Even if the noncontact sensors could give reliable information, the robot itself must be able to move to the exact positions specified. To move a fraction of an inch too close might damage the object or the robot, while moving a fraction of an inch less than required might not allow grasping and manipulation of the object. The required motion accuracy would require bulky, heavy, and expensive manipulators. Yet such accuracy can be achieved inexpensively and simply by means of contact sensors. Contact sensors mounted upon the end-effector itself and constantly monitored allow the robot to move (or the end-effector’s fingers to close) until contact is sensed, at which time motion can be halted. A relatively simple and inexpensive spring-loaded contact sensor could accomplish this, while a much more sophisticated and expensive system would be required
to accomplish this by means of noncontact sensors. To avoid crushing fragile objects, while applying sufficient force to not drop heavy objects, contact sensors (e.g., force) are an absolute must. Frictional forces between the end-effector and the object can be measured, and an appropriate force maintained while the object is grasped. Contact sensing is thus very useful and essential in manipulation tasks, which constitute a large proportion of robotic tasks not only today, but also in the foreseeable future.

1.4 Need for Multiple Sensors

For some robotic tasks, a variety of sensors may be required because any one sensor, or sensing modality, is limited in the amount and type of data it can provide. The limitations of one sensor can often be compensated for, however, by the use of another sensor [5]. For this reason, some robotic tasks may require more than one, or even a variety, of sensors. For example, the sensing of force in one direction is not responsive to object edges parallel to this direction. Sensing force in another direction, orthogonal to the first, will detect those object edges. Monocular vision alone does not provide adequate depth information. The use of two sensors, or at least two views, is required for the depth information provided by stereo vision. Even stereo vision does not provide adequate depth information for many purposes. Although range sensors do provide depth information quite well, they cannot detect patterns, textures, or differences in material, which are
made evident only by changes in color or intensity. Thus, vision and range are seen to complement each other. This is illustrated in the previous example, in which noncontact sensors provided the initial rough position approximation, while contact sensors were utilized in the final approach and making of contact.

There are also other reasons for the use of multiple sensors. One reason is the need for verification of questionable data. Another extremely important reason is the fact that sensor failure can and does occur. The use of redundant or multiple sensors can minimize the potentially devastating effect this could have on production in an assembly-line system. Another reason is to eliminate noise by means of averaging the output of several sensors. Many robotic applications involving manipulation require both noncontact and contact sensing modalities. It is often advantageous, therefore, to provide a robot with more than one sensor, and with more than one type of sensor.

1.5 Need for Sensor Fusion

The previous example of using vision to approximate the location of an object and then using contact sensing to establish its exact location entails the consecutive use of different sensing modalities. In some applications, the simultaneous use of differing sensing modalities may be required. For example, a mobile robot might simultaneously utilize both vision and ultrasonic range sensors to look for obstacles in its path.
Humans utilize various sensors both consecutively and simultaneously, and practice sensor fusion all the time. For example, while engaged in such a seemingly simple activity as walking, stereo vision is used to determine distance from obstacles, hearing is used to detect obstacles such as cars and other people, while a kinesthetic sense of joint position and sensing of contact between foot and ground are in use. Acceleration and orientation are sensed by the inner ear. All these sensory inputs are effectively fused by the central nervous system. Although humans do this without conscious thought or effort, it actually took a great deal of preparation. For example, a child must have three to five years of experience before being able to tie its shoes [2]. Today, an industrial robot is generally not provided with such sophisticated sensing capabilities by manufacturers [6]. Even those robots with special sensors do not have built in sensor fusion capabilities, which is one reason for their inability to perform some of the simplest tasks that humans perform routinely.

Humans accomplish sensor fusion by means of a not yet completely understood process utilizing their central nervous system, and which takes a relatively long time to fully develop. Robots, however, accomplish sensor fusion by means of other techniques. The determination and development of the most appropriate fusion technique or methodology are important issues in the science of robotics. As an example of a simple fusion technique, consider averaging the readings of two sensors. It can be seen that this technique is not appropriate for many applications. If the information to be fused comes from a vision sen-
Figure 1.1: Outline of fusion process.

Sensor and a range sensor, this technique would provide the arithmetic average of distance and brightness, information which is virtually meaningless. Thus, the fusion technique must be appropriately matched with the intended application.

The fusion process is outlined in Fig. 1.1. Now consider an example of sensor fusion. Consider the image shown in Fig. 1.2. This image shows the edge found by a sensor which measured the force applied in the horizontal direction (as seen in the image), as a robot probed the object shown in Fig. 1.3. Figure 1.4 was obtained by measuring the force in the vertical direction on the image.
Figure 1.2: Edge map obtained by applying Canny edge detector to force image constructed by measuring horizontal components of reaction force as robot probes object.
Figure 1.3: Object probed by robot – large valve handle.
Figure 1.4: *Edge map obtained by applying Canny edge detector to force image constructed by measuring vertical components of reaction force as robot probes object.*
Both Figs. 1.2 and 1.4 seem to indicate the presence of two openings or gaps in the circular outer rim of the object. However, the visual image shows that there are actually no such openings in the object. These apparent openings are a result of the sensing modality and do not reflect the true shape of the object. By means of sensor fusion [Fig. 1.5], shape information not present in either input force image has been restored in the output image. With this additional information it is possible to conclude that the contour of the outer rim is closed. Alternatively, it may be said that erroneous information (the presence of the gaps) has been eliminated by the fusion process.

Errors due to noise present in data can often be eliminated by means of fusion. A second sensor of the same type as the first can acquire data about the same object or scene. Data corresponding to actually existing features will probably be present in both images in corresponding locations (unless noise has completely eliminated them), while it is much less likely that noise present at a certain location in one image would also appear in the same location in the other image, since noise generally tends to be uncorrelated, while data obtained from existing features tends to be correlated. Noise or errors detected by one sensor and not detected by another sensor can be eliminated from the output.

One use of sensor fusion is to help identify objects by reducing the number of possible interpretations of data related to an object. In some cases knowledge about the sensing modalities used to acquire data can be used to eliminate interpretations not allowed by all modalities. For example, an object edge detected
Figure 1.5: Resultant image obtained by analytical fusion of horizontal and vertical force images.
by vision but not by range sensing can be eliminated.

Various sensors can be used to complement each other and provide a wider variety of types of information than possible from the use of a single sensor. This information must be fused, however, in order to be useful. Unlike the above examples, this requires the addition of information not found by another sensor, rather than the elimination of information. Different sensors also may present data of varying types, requiring different fusion techniques.

Sensor fusion is also necessary in the event of conflicting information from two or more sensors. For example, consider a mobile robot facing a wall. One sensor indicates to the robot that an opening exists at a certain location, while another sensor denies this. Some decision is necessary, even if it is to acquire further information, or to do nothing. This may be considered as one type of sensor fusion, since it involves producing some output to the robot based upon inputs from the sensors.

The evidence of the success of humans in autonomous manipulation and locomotion through the utilization and fusion of such various sensing modalities as hearing, pain, pressure, vision, etc. argues strongly for the potentialities of multiple sensing, contact sensing, and sensor fusion. This is especially true when one considers the relative inability of robots to perform these same functions as well as humans, if at all. It appears that there is a causative relationship between this relative awkwardness of the robots and their lack of multi-sensor fusion capabilities. In summary, multi-sensing and multi-sensor fusion capabilities are
necessary for the development of more useful, flexible, intelligent, autonomous, and sophisticated robots.

1.6 Synopsis

This thesis considers various sensor data fusion methods applied to force (contact) data. Chapter 2 is a review of work in the field of contact sensing. Chapter 3 is a review of work in the field of multi-sensor fusion. Chapter 4 presents sensor and data models and lists the desired characteristics of a fusion function. Chapter 5 considers in some detail 16 evidential reasoning based fusion functions, and how they compare to some of the desired characteristics introduced in Chapter 4. Chapter 6 outlines the development of a new analytical fusion function. Chapter 7 presents experimental results. Chapter 8 summarizes the thesis presentation and in particular compares the fusion methods considered and gives conclusions on this work. Recommendations for further research are also given.
CHAPTER 2

Previous Work in Contact Sensing

In this chapter, first some general types of contact sensors are discussed, including a comparison of human and robotic contact sensing. Some eminent work in the areas of contact transducer development, design of contact sensors, strategies for contact sensing, and processing of contact data are then considered.

Contact sensors are used to measure a variety of types of information. Some detect the presence or absence of contact, others measure the magnitude of the force, or pressure, applied normal to an object's surface, while others measure the tangential component of force. Other contact sensors utilize the combination of a specific geometry (e.g., a known internal lever arm) and force measurements to report the torque applied to the sensor. Still others detect and/or measure the degree of slip, or movement, of a grasped object. Some contact sensors measure acceleration [7]. Contact sensing elements can be arranged in a grid, or tactile arrays, and used to sense object shape. This is known as tactile sensing [8]. Most tactile sensors, or tactile arrays, detect or measure forces normal to the touched object's surface, at the point of contact; others, however, sense the tangential component of force (slip or shear), or torque [9,10,11]. An array typically yields a tactile image which can be processed by means of techniques similar to those
used in vision. *Simple touch sensors* can perhaps best be described as single elements from a tactile array, and usually detect or measure forces normal to the probed object’s surface, at the point of contact.

The contact sensors used with robots today are not nearly as advanced as the humans sense of touch. Most tactile arrays have a resolution of no more than a few hundred sensing elements at most, while the human skin contains an average of several thousand touch receptors per square inch [2]. The human skin contains a variety of types of sensors, such as acceleration sensors, pressure sensors, heat sensors, and pain sensors. While the material used in robotic contact sensors deteriorates with time, the human skin constantly renews itself.

Human touch is also *haptic*, while present robotic touch sensing is not. This means that, in addition to the *cutaneous* aspect, which robots have in their surface-mounted tactile arrays and humans have in sensing elements distributed over the skin’s surface, humans have a *kinesthetic* aspect to their sense of touch. This kinesthetic component refers to the integration of incoming signals from muscles and joints with control signals to the muscles. In robots this would refer to the integration of joint sensors and tactile arrays with actuator control signals. However, in robots, this kinesthetic component of touch has not yet been realized. Until recently, almost all robotic touch research has dealt with the cutaneous aspect only [12,13,14,15,16].

Much work remains to be done in the area of contact sensing. Of the three areas of robotics – mechanism, controls, and sensing –, sensing has historically
been the least developed. Contact sensing has lagged noticeably behind the areas of noncontact sensing, especially vision. Two major areas of contact sensing requiring more work are 1) device development and 2) control, information processing, and software issues [2,12,17].

Contact data can be very useful, but in order to be used, a transducer and a strategy for acquiring the data are required. Then, the data must be processed to obtain useful information. In the following sections, these topics are explored.

2.1 Contact Transducers

In this section, some of the devices used to sense force, torque, slip, and touch are examined.

Contact data can be acquired by means of various types of sensors. Simple touch sensing may be accomplished by various devices: the simplest is perhaps a switch which is closed by contact force and used to detect the presence or absence of an object between the fingers of the end-effector or for safety in the detection of obstacles [12,18,19]. Other types of touch sensors include those based on the Hall effect [20], hydraulic sensors [12], and pneumatic hole-to-hole alignment sensors [21]. In applications where there is a need to sense contact without moving or disturbing a very light and/or fragile object, a simple but non-intrusive pneumatic whisker sensor can be used [12,22]. The whisker is a thin hair-like device which does not disturb most objects upon contact. Whisker
motion due to contact can be sensed in many ways, including optically and pneumatically.

Contact can also be detected by optical means. For example, contact can cause a moveable spring-loaded rod to obstruct the path of light passing from a photoemitter (e.g., infrared light emitting diode) to a photodetector (e.g., phototransistor) [23]. The magnitude of the force applied can be measured by using an array of photodetectors instead of one, since the degree of motion of the rod, and thus the number of photodetectors blocked, will be proportional to the force applied [12].

Piezoelectric materials, which produce a voltage when stressed, can be used for simple touch sensing as well as analog pressure measurement. Quartz, although the best piezoelectric material, is not very adaptable to tactile sensor packaging. Polyvinylidene fluoride (PVDF), however, is a very useful piezoelectric material for these sensors [12, 24, 25].

Magnetoelastic materials, which change either their length (Villari effect) or their flux density (magnetostrictive effect) as a result of an externally applied force, have been used in torque and slip sensing [26]. Some materials exhibit both effects when made into long thin rods or ribbons, although they may be brittle [24, 27]. Some advantages of magnetoelastic materials are their excellent sensitivity, low hysteresis, durability, corrosion resistance, simple circuitry, wide range of operation (0.004 psi at 266 psi), low voltage, and high power output. Although magnetoelastic materials are sensitive to stray fields and to voltage
changes, this can often be solved by shielding, filtering, or a second set of symmetrical windings to cancel errors.

Electromagnetic, semiconductor, and piezoelectric transducers have also been used to detect slip via a workpiece–contacting stylus. For example, a knurled roller has been used between the stylus and the workpiece [23]. The vibration of the stylus is detected when slip occurs. Slip–induced motion of a rolling ball has been detected by means of discrete magnetic or optical markings [12,23]. If the ball rolls on an axle, slip can be detected in only one direction. Recently, the detection of both direction and amount of slip has been achieved [28,29,30]. Acceleration sensors have been utilized to detect slip, guided by evidence that humans also use them to detect slip [7]. Many types of transducers for tactile arrays have been developed. One simple type is a bistable dome which snaps into a stable position and establishes an electrical contact upon object contact [12,22]. Many transducers used in simple touch can also be used in tactile arrays, and vice versa, since a tactile sensor typically consists of a matrix of individual simple touch sensors [31,32,33]. Common types include piezoresistive, resistive, and semiconductive sensors and those employing conductive, compressible, and flexible materials [34]. Hydraulic, capacitive, piezoelectric, optical, and electromagnetic sensors are less common [35,36,37]. The utility of tactile arrays is illustrated by the fact that tactile arrays of no more than eight elements (which would sense slip or shear) have been found to be adequate for 80% of the tasks suitable for robotics at six manufacturing plants [12]. Many of
these sensors, however, suffer from disadvantages such as: large number of wires, lack of physical robustness, low dynamic range outputs, and noisy outputs [38]. One method of detecting contact and its location that does not use the typical tactile array is described by Bastuschek [39]. In this sensor, the upper layer is conductive, while the lower is resistive. Pressure establishes contact between these two layers, allowing current to flow. This current is measured at the four sides of the resistive layer. These four values allow the computation of the location of the point of contact. One advantage of this method is the small number of wires required.

Conductive elastomers have been found to be useful in array tactile sensors; however, they deteriorate over time [39]. They are often used in the form of rods stacked in two perpendicular layers [40]. A voltage is applied across the two layers. When a compressive force is exerted on the sensor, the area of contact between the layers increases, resulting in a higher flow of current. The output of such a sensor is logarithmic, allowing a wide force range and accurate readings of small changes under both light and heavy loads. Conductive plastic and conductive foam have also been used in this fashion [12,40,41]. Capacitive touch sensing elements have been used in tactile arrays by many researchers [42,43,44,45,46]. In one such sensor, described by Siegel et al. [47], an applied force compresses a dielectric, which changes the value of capacitance. The sensor consists of an array of capacitive sensing elements. One capacitor plate of each sensing element is connected to all other elements in the same row, while the other plate is con-
nected to all elements in the same column. Rather than each element having its own detector, touch is detected by amplifying and reading the column outputs as a signal is applied to each row. This scanning technique reduces the number of wires required. Other types of capacitive touch sensors use electrorheological fluids as dielectrics. The electrorheological fluid allows the sensor to conform to the shape of an object. Then, when a potential difference is applied across the fluid, it solidifies, thus allowing a firmer grasp on objects and preventing slippage. Kenaley [48] has described one such sensor. Some of the disadvantages of capacitive sensors include their size, strong materials dependence, possible influence by external fields, and inadequate sensitivity and reliability [12,47].

A tactile array can also utilize reflective surfaces that deform upon applied pressure. The reflected light is transmitted through optical fibers to an array of photodetectors [49]. Also, a light guide may be used between a CCD array and a flexible cover. Light sent through the guide is normally totally reflected, but an applied force causes light to scatter and be detected by the CCD layer [23]. When optical signals produced by contact are carried to photodetectors by optical fibers, flexibility in the physical design of the sensor is allowed. The separation of the sensing elements from the electronics is possible, making electronic shielding easier [17]. Optical techniques are free from the electromagnetic interference which can degrade the performance of capacitive sensors and sensors utilizing magnetoresistive or magnetoelastic materials. Other advantages of fiber-optics are its long life and simple, tough construction [35,50]. However, they suffer from
inadequate time response, limited dynamic range, and lack of robustness [12,51]. Further materials development may render optical methods more attractive in the future.

VLSI technology has alleviated some of the communication problems in tactile data sensing. Some of the data processing can be done on the same chip as the sensing elements themselves, thus speeding up communication and reducing the information into fewer bits, thereby reducing the number of wires required [52,53]. The data reduction, pre-processing, and easy multiplexing allowed by this approach are advantageous for many applications. Including computation elements on the VLSI sensor itself can lead to fewer wires in the system, low power consumption, wide signal bandwidth, and reduced weight. Because of the small size of the sensing elements, such a sensor can have a high resolution [24,54,55]. One major disadvantage of such systems, however, is their lack of robustness in harsh industrial environments [12,56].

Artificial skins consist of a tactile array underneath a flexible protective shielding [23]. When pressure is applied to the sensor, the cover is pulled down in the neighboring area. Thus, some sensing elements which are not directly underneath the point of contact may be activated, producing a “fringe” effect. This may be desirable for sensing contact at points not directly above any sensing element, but, on the other hand, often produces an undesirable blurring effect in tactile images [40].

Conductive membranes are sometimes used as skins. In one configuration,
the top layer is a membrane of silicone rubber with nonconductive slices sandwiched between the conductive slices to allow electrical conduction in one direction only [53]. The bottom layer conducts only at a right angle to the direction of conduction of the top layer. A middle layer allows contact between the other two layers only when pressure is applied. The output is scanned by applying a potential difference to each column, one at a time, and measuring the current flowing in each row, thus reducing the required number of wires leading from each sensing site and from the sensor itself. Some of the difficulties encountered in using conductive membranes include noise, fatigue, low sensitivity, long time constants, hysteresis, nonlinearity, and drift [12,19,24].

The National Institute of Standards and Technology (previously National Bureau of Standards) has developed a magnetoresistive skin [53,57] in which the conductivity changes under the influence of an external magnetic field. In this sensor, magnetoresistive elements were etched onto an aluminum oxide substrate and separated from a copper strip directly above it by a compressible insulating layer. When a current was pulsed through the copper strips, the corresponding magnetoresistive strip was exposed to a magnetic field which changed its conductivity, dependent upon the variation in pressure applied to the insulating layer [24,27,57]. Another magnetoresistive tactile array has been developed to measure shear (slip), torque, and normal forces by sensing the position and orientation of magnetic dipoles in an elastic medium. However, this sensor lacked ruggedness, suffered from hysteresis, and was susceptible to magnetic objects
and fields [12,58].

In force and torque sensing there are four general approaches used. The first is passive, or open-loop, compliance. This provides no information, but allows the end-effector tool to make adjustments for minor position and orientation errors, and is roughly analogous to the use of chamfers to guide the insertion of a peg into a hole. The compliance is mechanical. One example of such a device is the Remote Center Compliance (RCC) mechanism, developed at Draper Labs [59,60]. A second method is the sensing of the resultant force at the manipulator arm joints. This can be done by measuring the armature current for electric motor-controlled manipulators, or the fluid pressure for hydraulic- or pneumatic-controlled manipulators, and generally yields only one component of force in the direction of motion at the involved axis. The sensing of more components of force (or torque) is allowed by the third method, that of sensing reaction forces at a pedestal. These forces can be better localized, however, by the fourth approach, the use of a wrist-mounted force/torque sensor.

Devices used to measure force include electrical contact switches, strain gauges, and devices which measure either the current of electric actuator motors or the air pressure in pneumatic actuators. Strain gauges are the most common force/torque transducers in use. They are most often used in wrist-mounted force/torque sensors [23].

A typical strain gauge consists of some wire-like structure which changes its electrical resistance upon deformation. A strain produces deformation of the
structure, resulting in a slight change in the length of some dimension of the structure. Wire is wrapped around the structure so that the wire is stretched when a strain is applied. Several loops of wire are used to increase this effect. The result is a measurable change in the resistance of the wire, indicative of the magnitude of the applied force. Several loops of wire can be oriented in several different directions to allow sensing of all components of an applied force [23]. Six transducers is the minimum number required to provide three components of force and three components of torque, although many configurations use more than six transducers [35,61,62]. Since strain gauges are sensitive to temperature, some kind of temperature compensation is often incorporated into their design.

Some strain gauges operate by means of a change in resistance which is not due to a change in length, but directly due to strain itself. These are known as piezoresistive strain gauges. Silicon is piezoresistive, and can be shaped and made into thin membranes which produce electrical signals upon strain. It is versatile, linear, reliable, and sensitive, yet expensive. Piezoresistive strain gauges are compatible with silicon technology and show promise, yet suffer from the disadvantages of being stiff, flat, and slippery [24].

Bonneville Scientific Corporation [63] has developed an ultrasonic pulse echo technique for measuring force [23]. The time required for an emitted ultrasonic pulse to travel back and forth through the pad material is used to measure the pad's thickness, which varies as a function of the applied force or pressure. The transducer which generates and receives the ultrasonic waves is usually a piezo-
electric material, such as polyvinylidene-flouride (PVDF). An advantage of this method is that since the elastic element need not be conductive, the desired mechanical characteristics can be better optimized by the choice of material [24,63].

2.2 Design of Contact Sensors

To achieve desirable contact sensor properties, precise guidelines and design criteria are necessary [64]. Yet in the past, the design of contact sensors has been largely ad hoc. In the following, some of the desirable characteristics and properties, as well as design considerations, of touch sensors are discussed.

In general, since sensors are a crucial interface between the robot and its environment, the first and primary consideration in their design must be the environment in which the sensor is to be deployed [17]. This can give a clear idea of the desired sensor characteristics, which in turn should guide the direction of sensor design efforts. More specific design requirements are given below.

Tactile arrays are commonly installed at the end-effector level of a robot because other sensors, such as strain gauges and pressure transducers, though very sensitive, are too fragile, large, and heavy [65]. The ideal characteristics of tactile arrays are as follows [12,66,67]: (1) spatial resolution: 1 – 2 mm, (2) number of sensing elements: 10 × 10 (most researchers suggest a range of 50 – 200 sensing elements), (3) sensitivity: 1 – 1000 g, (4) dynamic range: 1000:1, (5) hysteresis: low, (6) response time: 1 – 10 msec, (7) skin material: robust, for
harsh industrial environments, (8) the sensor–containing surface be deformable, (9) the sensor be mechanically rugged and reliable, (10) it use a small number of wires, (11) there be uniformity among the individual elements of an array in their sensitivities.

The equalization of transducer sensitivity has been found to be even more important than increasing the sensitivity of only some elements, which could make accuracy worse [60]. Hysteresis may not be totally undesirable. It may be tolerated if the robot has recorded the time sequence of its moves, knows the shape of the hysteresis curve, and has a mechanism of determining whether the applied force is decreasing or increasing [47].

According to Hackwood and Beni [17], there are two function–based classifications of contact sensors. The first type, shape sensors, are used primarily for shape detection. The second type are manipulation sensors which are used primarily to measure relative forces between object and end–effector.

First, consider shape sensors. In trying to sense the shape of an object, a slipping, sliding, and/or rotating object would result in problems with the interpretation of the contact data, roughly analogous to the problems caused by blurring in a vision image. Thus, global sensing of torque and slip is required. Any shape must be described by more than a single point. Lines require two points, planes at least three (noncolinear) points, and more complex shapes require even more points for their description. Even though the inclusion of orientation information with the points, as provided by surface normals, reduces
the number of points required, a single point does not provide sufficient information for sensing any shape more complex than a plane. Thus, for shape sensing, local sensing of normal forces, as well as global sensing of torque and slip is necessary. Therefore, shape sensing requires much more than a single sensed point. One method of obtaining this information is by repeatedly probing the object to be sensed at different points. However, this method is too time-consuming for many applications. A faster method involves the use of an array of several sensing elements, or a tactile array, which gathers information about many contact points virtually simultaneously.

In manipulating sensors, tactile arrays may also be used. In manipulating fragile objects, careful control of end-effector forces is required [68,69,70,71]. There is a need for sensors that can provide accurate information about these forces. While simple pressure transducers or binary touch sensors may suffice to detect failure or success of mere grasping and holding, manipulation requires detailed monitoring of forces, since the manipulated object is often temporarily out of equilibrium. While joint sensors alone are insufficient for this task, two methods suffice. The first is the use of tactile arrays. Pertinent design considerations are considered below. Another method, known as Intrinsic Tactile Sensing (ITS), is also described below.

In tactile arrays, a very important design issue is, what type and proportion of elements should be used for a manipulation sensor? There are three types of array elements: (1) those that sense normal force, (2) those that sense torque,
and (3) those that sense slip (shear). The force sensing type is useful primarily to indicate the presence or absence of an object. This can be accomplished, however, with a single global sensor; thus, this type of sensing element need not be incorporated into an array. Slip (shear) sensing elements can detect both slip and torque, while torque sensing elements detect torque only. Therefore, slip sensing elements are the most important type for a manipulation tactile array [17,72].

The appropriate placement of sensing elements in an array is another important design issue. It should take into consideration not only the relative positions of the sensing elements with respect to each other, but also the number of sensing elements and their mobility. Six important factors to be considered in sensor arrangement are the size of the sensing sites, the number of sensing elements, the speed of motion of the elements, the response time, the processing time, and the area to be sensed. The proper choice of the sensing element arrangement requires careful consideration of all six variables. For example, for sensing an object smaller than the size of each sensing element, a different arrangement is called for than that required for sensing an object larger than the size of each sensing element [17].

As previously mentioned, tactile arrays can provide information required by manipulation sensors, but they are more than sufficient [38,40,73]. Intrinsic Tactile Sensing (ITS), however, requires fewer wires and involves simpler hardware than tactile arrays and yet is adequate for use in manipulation. It utilizes
a force/torque sensor usually mounted inside the end-effector fingertip. ITS applies geometric calculations with pure force and torque data, as opposed to the extrinsic approach, which measures externally applied forces (often only the presence or absence of a force) at external points of contact on surface-mounted arrays. The intrinsic method would be useful to measure overall contact conditions, while the extrinsic method would be useful in the detection of local object features. The extrinsic method can sense large areas of contact and produce tactile images [17,61].

The intrinsic approach has the advantages of [38,74]: higher bandwidth, theoretically infinite spatial resolution, faster, more accurate contact, measurement with freedom from hysteresis and non-linearities, fewer wires, detection of slippage, measurement of friction, robustness, speed, and reliability. It can find normal contact force intensity, the position of contact on the end-effector, the intensity and direction of tangential (frictional) force, the intensity of friction-generated torque, and can find this information rapidly by means of closed form algorithms when the finger shape is ellipsoidal. It can monitor contact forces in real-time, and with a proper model of friction, prevent slippage. ITS provides more useful information about contact stability than any other currently available sensor.

Force sensor design is very important in ITS systems. Since the appeal of simplicity, low cost, and limited number of wires in force sensors suggests the use of the minimal configuration — six strain gauges — and since work
thus far has dealt mostly with extended configurations, Bicchi and Dario [61]
investigated optimal transducer placement for the minimal configuration. A six-
element vector \( (M) \) representing the outputs of the strain gauges can be related
to the six-element vector \( (F) \) representing the actual applied forces by a sensor
compliance matrix \( (C) \) where \( M = CF \). The elements of \( C \) are functions of
the transducers themselves and their geometric arrangement. The "condition
number" of the matrix \( C \) is defined as the product of the spectral norm of \( C \)
with the spectral norm of its inverse. Calibration errors are proportional to this
number, so one method of optimizing placement of strain gauges is to minimize
this number [61]. The use of the condition number, however, is not without its
disadvantages [75]. Although sensitivity may be improved by multiplying the
matrix \( C \) by the appropriate scalar, the condition number would not indicate
this improvement. A redundant transducer's effect is not correctly indicated. If
the number of transducers is greater than six, the condition number offers no
information about relative errors. When the number of transducers is greater
than six, there exists a variety of inverses for the \( C \) matrix, about which the
condition number gives no information.

Therefore, three design criteria have been suggested to evaluate the elastic
components of force sensors [75]: (1) Strain gauge sensitivity: this is the Eu-
clidean norm of each row of the matrix \( C \). It should be close to, but less than,
unity for each row. (2) Force sensitivity: the effective force output in all six (3)
Minimum stiffness: for the most compliant direction, this should be as large as
possible.

The above criteria are by no means complete. Other factors, such as weight and the ease of parts machining, should be considered also. A sensor utilizing these criteria has been designed, as well as the hardware for performing part of the computation of the inverse of the sensor compliance matrix [75]. This design used 24 strain gauges and six Wheatstone bridges to compute forces and torques applied to the end-effector and would be useful where a high sampling frequency is required.

Intimately related to the design of contact sensors is the design of fingers, end-effectors, and manipulators upon which they are mounted. This affects the relative location of sensors as well as the objects to be sensed [76,77,78,79,80].

2.3 Sensing Strategies

Of all human senses, the most active and dynamic is touch [81]. Active touch sensing can be defined as the utilization of deliberate sensor movement and strategies to collect contact information [82]. When humans search in the environment for contact information, such as when they grope for a light switch in a dark room, they exhibit active touch sensing. Since active touch sensing in humans is so successful in performing tasks presently impossible or very difficult for robots, the implication is that active contact sensing is also important for robots [12,83]. For example, active control of internal grasping forces can pre-
vent slippage by ensuring sufficient friction [38]. Other methods of controlling gripping force include vacuum operated rubber snouts, magnetic grippers, segmented structures containing internal cables, and multiple vacuum fingers [12]. One use of active touch would be for a robot to probe an object. The robot would stop motion upon contact (force exceeding a threshold). The position, as well as the reaction force components, could be stored as images. The information thus obtained could be used for recognition, or to obtain detailed shape information [84]. The major drawback of this method is the length of time required to collect the data. In circumstances where time is not a constraint, it can be very useful. In contrast with active touch sensing, passive touch sensing involves no sensor movement. Passive touch sensing is of little utility, and is used on rare occasions. Although early research focused on passive recognition of objects placed on a tactile array, active sensing is now mandatory for exploring robots [12,17,85,86,87,88,89].

One possible application of active contact sensing is in the creation of contact images of objects by means of a single force sensor, instead of a tactile array. This would obviously be useful where a tactile array is not available, but also where an array would not be useful due to inappropriate object shape. For example, if the surface of the object to be contacted had important features at several different widely-separated levels of depth, or at the bottom of relatively deep, narrow cavities, a tactile array would provide information concerning only the outermost part of the object's surface, due to the fact that the array typically has

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a relatively rigid planar construction. However, the recessed areas and cavities
could be probed by a rod-like tool. A single force (or contact) sensor could be
utilized to detect contact and halt motion. Then, the position of the probe could
be recorded.

However, for truly sophisticated robots, it is obviously desired to maximize
autonomy and versatility. Therefore, more is required of the sensing process
than that it be merely active. Rather than moving to and sensing at a number
of pre-programmed positions, it is desirable that a truly advanced robot be able
explore its environment, and intelligently modify its actions on the basis of its
discoveries in order to obtain the most useful, relevant data with the fewest
possible moves. In other words, the robot should be able to intelligently “look
for” useful contact data [90]; it should continuously modify where it probes based
upon the incoming data. This would free human programmers from having to
specify each move and allow the robot to operate in an environment with fewer
constraints, thus increasing the flexibility and usefulness of the robot. This
efficient active gathering of sensed data is referred to as intelligent machine
perception [17], and is one of the two fundamental areas of sensor science. The
other area is pattern analysis. So far, the emphasis in sensor science has been
on pattern analysis, but developing strategies for the robot to use in gathering
contact data efficiently is equally, if not more, important [19,38,40,91].

Yet some work in intelligent machine perception has been done. Schneiter
and Ellis [92] have devised a strategy employing intelligent machine perception
for object recognition and location by a robot. They assumed the sensed object
to be planar (2-D) and polygonal, but the method they described is expandable
to three dimensions. They assumed that the robot could detect the position of
the point of contact between its hand and an object, and the orientation of the
surface contacted (by means of sensing the direction of the normal force at the
surface). They also assumed that the robot had access to models of the objects
to be manipulated [93]. For such a robot, the first move must be a blind one.
However, after initial contact is made with an object, there are a finite number
of possible identities of the object, as well as a finite number of orientations and
positions corresponding to each possible identification. Continued measurements
will reduce the number of possible interpretations, until only one (the correct
one) or none, is reached. If no possible interpretation fits, the object cannot be
identified. The major cause of failure of their approach to correctly identify an
object was measurement errors [94,95,96,97,98,99].

Cameron [72] has utilized statistical decision theory to determine optimal
sensor placement, or moves, based upon previously sensed object information
and object models. The next sensor position, or move, is chosen which will
allow the maximum number of object interpretations (identification, location,
and orientation) to be eliminated with the fewest successive moves. This method
can be used with noncontact sensors, and in fact was implemented with a vision
sensor. Like the previous technique, this one operates upon two-dimensional
objects, but is expandable to three dimensions.
2.4 Processing of Contact Data

Tactile array sensors produce images similar to those generated by vision sensors. Just as vision image picture elements are commonly known as pixels, tactile image elements are commonly referred to as taxels. Tactile images are used to recognize objects, as are vision images. Therefore, it should come as no surprise that pattern recognition and image processing techniques useful for vision images are also useful for contact images [19,40,91,100,101]. Vision algorithms have been successfully used for acquisition, recognition, and manipulation with tactile array sensors of varying resolutions. Vision techniques which have been successfully applied to contact images include median filtering, thresholding, gradient operators, histogramming, template matching, region growing, edge detection, and clustering [91,102,103,104,105,106]. Also, perimeter and area have been used to recognize and to discriminate between objects by means of algorithms similar to those used in vision. The orientation of objects in contact images produced by a 16 × 16 array has been found using moment techniques similar to those used in vision to within only one degree, which is sufficient accuracy for many robotic applications. The center of an object (center of gravity) also may be found by vision algorithms using moment techniques [107,108,109]. The surface texture of an object may be perceived by touch [110,111].

Circles, rectangles, and many other shapes have been successfully recognized with 5 × 10 and 10 × 10 tactile arrays [12]. Most objects recognized in these
studies have been simple in shape, because the complexity of the recognition algorithms increases as the similarity in shape between the objects to be recognized increases. Unfortunately, methods which work for such simple shapes may not work for other more complex shapes [19]. Most automatic pattern recognition results have been rather simple. Parallel digital algorithms dealing with complex 2– or 3–D shapes are practically nonexistent for both vision and contact images. One of the reasons for the slow progress in contact pattern recognition has been the lack of use of parallel processing [12]. Briot [112], however, has developed a recognition technique using a syntactic approach, which works on any shape object. His method is very useful for manipulation, which he showed to require only a few object features [19,113,114].

In contact pattern recognition, sensed objects may be divided into three categories. First, the object may be smaller than the sensor array. In such a case, the object may be identified by its texture [12]. Second, the object size may be commensurate with “hand” size. In this case, the object may be grasped and use made of the fingertip positions and joint angles, rather than array resolution, to recognize the object. Finally, if the object is large compared to the end–effector, active exploration is necessary.

The complexity of the pattern recognition techniques required is a function of sensor noise, the number of objects to be recognized, and their similarity to each other. The complexity also increases as the sensor–based information becomes greater than the initially–known information [19,104,115,116].
CHAPTER 3

Previous Work in Fusion of Multi-sensory Data

The data acquired by various contact sensors, and/or other noncontact sensors, can often be combined in such a way as to provide information not available from either sensor alone. This process is known as the fusion of multi-sensory data. In this chapter, first, general methods of data fusion will be considered, and then the more specific area of multi-sensor fusion.

3.1 Historical Trends in Data Fusion

There exist a number of procedures for fusing knowledge about the existence or nonexistence of a feature in a scene. This inference problem also can be regarded as a mechanism of updating knowledge about a given event. Assume that sensor $S_1$ has the knowledge or the belief of the existence of a certain feature. The availability of a new measurement given by sensor $S_2$ will lead sensor $S_1$ to update its original assessment because of the new knowledge acquired. This is a general problem in which information acquired by one sensor needs to be updated because of the presence of additional information provided by another sensor [117,118].

There are four basic categories of methods for inferring knowledge from two
(or more) sensors. The first class of techniques is based on the *Super-Bayesian theory* [119]. These techniques are centered around Bayes’ theorem which uses past knowledge about the occurrence of an event to infer the chances of occurrence of that event in the future. The second class of techniques is based on *Belief (or Evidence) Theory* [120]. These techniques use Dempster’s rule of evidence combination [121] where the belief in the occurrence of a given event is computed as a function of two or more assessments provided by different knowledge sources. A third class of inference mechanisms includes those based on evidential reasoning functions often defined in a fuzzy framework [122,123]. Various evidential reasoning functions have been proposed over the years. These functions are simple two-dimensional functions of the form $r = f(p, q)$ where $p$ ($q$) is the assessment of sensor $S_1$ ($S_2$) about the occurrence of an event $A$, and $r$ is the proposed combination of $p$ and $q$. The fourth class of techniques includes methods that do not fit either of the three categories mentioned above. These techniques infer knowledge from two assessments by using some constraints on the evidence collected to infer the final judgement. They are often known as analytic or geometric techniques. In the following, the first three categories are briefly discussed. In Chapter 6 is outlined a new fusion technique which overcomes some of the difficulties in many application areas of interest.

Combining evidence within a *group decision* framework can be done through probabilistic methods; *Bayes’ Theory* [124] is the most known in this category. To many probabilists, however, Bayes’ theory has serious problems in combin-
ing evidence. Assumptions regarding informative priors produce unfair fusion between the priors and the likelihoods, especially when priors dictate the final outcome of the fusion. Representation of ignorance (of a sensor) results in severe inconsistencies as well. Often, these approaches cannot distinguish between a lack of belief and disbelief. Bayesian decision making schemes assume that the assessment makers or knowledge sources are always coherent. This assumption is acceptable if the assessment makers are human, as is the case in most systems which support the use of Bayesian approaches. However, for machine-driven systems, incomplete or inconsistent data generated by sensors is encountered systematically and needs to be dealt with accordingly. For most practical applications, the lack of priors or the availability of noninformative priors presents a significant obstacle toward the use of Bayes’ theory to perform sensor fusion. Noninformative priors usually rely on experimental data which may violate the principle of likelihood. In addition, evidence combination performed using noninformative priors has often resulted in poor performance typically reflected in an assessment that contradicts either or both of the original sensor assessments.

One of the first non-probabilistic or non-Bayesian approaches in dealing with evidence combination in data fusion is the Belief Theory developed by Dempster and Shafer. As an alternative to the theory of probability, the theory of belief deals with weight, evidence, and an assigned degree of support reflecting a belief. This theory is based on the premise that each source of information provides only a partial belief about a proposition. The total belief is best deduced by

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using the Dempster’s rule of combination which pools fairly different sources of information before a final assessment is made. From a statistical point of view, the theory of evidence is not problem-free. In general, this theory produces an overestimation of the final assessment; this is undesirable in most machine applications. Other problems arise in using Dempster’s rule of combination as a result of the sensitivity of such a rule to numerical variations in the inputs. In other words, a small change in the input can cause a drastic change in the output. Though sometimes recognized as a viable tool to combine conflicting bodies of evidence, there are still some weaknesses in doing so in many situations. This method performs relatively well when the proposed bodies of evidence are in relative agreement. In contrast, its performance is very poor in the presence of sharply conflicting bodies of evidence.

The theory of evidence is not the only non-probabilistic method for solving disputes between sources of information. Fuzzy sets [125] deals with inexact, incomplete, or uncertain quantities. The need to deal with real-world problems requires more general settings which precise mathematics — with its usual ideal assumptions — is unable to handle. The reason is that most statistical problems are ill-posed; a fuzzified version of these concepts is a more effective tool in dealing with decision making processes. Under the fuzzy framework, Possibility Theory [126] has emerged to represent imprecision in terms of fuzzy sets and to quantify uncertainty through four proposed notions: possibility, necessity, plausibility, and credibility distributions. In the context of fuzzy mathematics,
aggregation operators such as “min” and “max” have been found to be equivalent to the “intersection” and “union” operations in the traditional set theory mathematics. In addition, many more approximate reasoning operations have been proposed and tested in various areas of application.

In fuzzy theory, instead of an element being either a member or a nonmember of a set, an element can have varying degrees of membership. The range of values is not binary. It may even be continuous. For example, a membership value of 1 would correspond to an element being a member of a set in the classical sense, while a membership value of 0 would correspond to the classical notion of an element not being a member of a set. However, fuzzy set theory allows a membership value of 0.8, for instance. The interpretation given to information to be fused may be that of a fuzzy membership function. For example, the strength of an edge in an edge map may be indicated by the intensity of the pixels on the edge. A high value could be interpreted as a high probability that a pixel lies on an edge, or as a high value of a membership function with respect to the set of edges. These probabilities or membership functions can be fused by various techniques, including ones based on evidential reasoning, considered in more detail in Chapter 5, and a new technique to be examined in Chapter 6.
3.2 Sensor Fusion in Robotics

One of the latest reports on the state-of-the-art of robotic multisensor fusion may be found in [5]. This reference is a unique source in terms of depth and variety. In the following, a brief discussion of the information given in [5] is presented.

Allen [127] proposed a strategy to organize sensors and sensory related tasks as classes and methods in an object-oriented programming system. His goal is to avoid changing the system design as the task changes, in order to yield an expandable framework to handle any other suggested tasks. He also described a framework for implementing a multisensory system in two proposed logical high level virtual sensors: an intelligent hand for grasping and a mobile eye used for motion tracking. The intelligent hand utilizes a tactile sensor and finger servos to control the coordination of grasping and tactile recognition. The mobile eye utilizes a vision camera. The main idea of his discussion is to relate sensing to actuation by treating these entities as objects in an object-oriented programming system. His idea of an object-oriented system is a set of entities (objects) that communicate arbitrarily via messages that allow object methods to be invoked depending on the message content. Several level classes that utilize more than one sensor are developed. For instance, lower level sensor classes seek to find a point of contact based on texture, focus, or some other vision operator. Another level seeks to find edges using both vision and tactile
sensors. Other sensor classes identify surfaces which can be identified by other sensors. In addition, if a new device is very different from the working devices, a new sensor class or possibly new methods can be added to the overall system, since it can be expanded for new tasks and new sensors. An example of locating and grasping disks is given. First, stereo vision is employed to locate the three-dimensional position of the disks. Then, using the tactile sensors on the hand, the fingers are able to precisely locate and grasp the disks. The tactile sensor would compensate for the error that could occur due to the vision system’s inaccuracies.

Ayache and Faugeras [128] described a method for developing a three-dimensional description of the environment of a mobile robot using stereo vision. Different descriptions of the environment, obtained from sometimes unknown or imperfectly known positions, are fused. This methodology makes use of the extended Kalman filter to deal with noise. Explicit representation of noise in the world model along with the use of multisensory data is recommended to deal with the noise problem. From every position at which a stereo pair is taken, geometric primitives such as points, lines, and planes are extracted. Also, the uncertainty corresponding to these primitives is expressed in the form of a set of covariance matrices. Each description is attached to a local coordinate frame. When the translation between two local frames is imperfectly known, or unknown, the same geometric primitive(s) can sometimes be found in both descriptions of the environment attached to these frames. Matching these prim-
itives yields a better estimate of the three-dimensional transformation between frames, which in turn allows a better estimate of the position and orientation of the geometric primitives. The description of the environment is thus updated at each local frame for those geometric primitives found in other descriptions attached to other local frames as well. This methodology has been successfully implemented by the authors on real stereo pairs.

Durrant-Whyte [129] proposed a general framework for modeling observations and decision making under the assumption of uncertain geometric information. He described a technique for modeling sensors and their observations. Every sensor in the system is treated as an individual decision maker which can communicate with other sensors. All sensors in the system are sources of many uncertain pieces of information that need to be fused to form a consensus and to optimize decision making. Three sensor models are combined to generate the complete sensor model. First is the observation model, which is essentially a model of sensor noise and error. Second is the dependence model, which describes the effect of other sensor observations on sensor measurements. Third is the state model, which describes how the position and the internal state of the sensor affects measurements. The drastic simplification in dealing with fusion if one chooses to use nondeterministic approaches such as probability theory is emphasized. Many multisensory data systems utilize various sensors which generate different kinds of data that need to be fused. These types of data are hard to fuse unless some transformation to a common domain takes place for all of them.
Such transformations are important because they allow all different sensors to interact and communicate in a dimensionless language. This common language is derived by transforming all data to probabilistic models. Also it is necessary to have quantitative models that allow for analyzing the system’s performance and ability of estimation, and for developing sensing strategies to serve the task requirements. In contrast, heuristic rule-based approaches are unable to quantify sensor performance. Once data are transformed to a common parametric domain, opinions of sensors are fused using a Bayesian classifier. Contaminated Gaussian distributions are proposed to represent observation models; they are capable of handling not only information contaminated by random noise, but also information which contains systematic noise as well.

Flynn [130] proposed a system incorporating a combination of sonar and infrared sensors for mobile robot navigation. This system takes advantage of the best characteristics of each sensor. For example, sonar sensors have a low angular resolution; however, they are capable of providing accurate depth measurements. Infrared sensors, on the other hand, have good angular resolution, but are not able to measure depth accurately. Infrared sensors are more useful in detecting objects. In this work, a refined map is created by combining data from two sensors. This is done by choosing which sensor to believe when they disagree. This choice is is based on sensor characteristics. For example, the sonar has a greater range than the infrared sensor. Consequently, beyond its range of operation, the infrared sensor reading is ignored. Also, when the
infrared sensor detects a sharp change from detection to no detection over a relatively small area, while the sonar detects no such change, the sonar is ignored. This is because the sonar beam width is so large that small openings such as doorways are sometimes not detected. This map is then converted into an intermediate representation of the sensed data containing features, such as the curvature primal sketch. Knot points are placed at points of greatest curvature and then connected by straight line segments. These knot points are based on local features. This is advantageous because when using several different views of a room, the global shape can change greatly. Local changes, however, vary less, especially for incremental changes in robot position. Path planners usually require a list of polygons as their input. These can be generated from the curvature primal sketch by expanding the line segments between knot points into thin rectangles. Then, a global map can be generated by merging curvature primal sketches obtained from different positions of the robot. In this merging, knowledge about whether edges present in the input sketches were actually detected, or were added as filler points for the curvature primal sketch algorithm, is used to decide which edges are to be kept from which images, and which are to be deleted. This method provides an autonomous robot with intelligent path planning in industrial environments.

Henderson et al. [131] described a Multisensor Knowledge System (MKS) utilizing computational units tuned to special, commonly occurring configurations of lines, such as perpendicular and parallel lines. These units are implemented
as *Logical Sensors*, which receive commands from a higher level controller and other logical sensors. They choose, on the basis of these commands, which of several programs to use. This MKS contains a frame–like knowledge base to describe diverse system components, from control algorithms to actuators. This is needed to allow greater flexibility, ease, and speed in multisensor system design to meet the needs of manufacturing and defense applications. Ease and speed of interfacing one component to a perhaps radically different component is needed. The MKS supports (1) multisensor system specification such as components and interconnections, (2) sensor, algorithm, processor, and actuator knowledge representation, and (3) multisensor system simulation in order to evaluate system performance. System components are described in a hierarchical manner; e.g., sensor, vision sensor, camera, CCD camera, Fairchild CCD camera. Sensor features, such as error and drift, can be put into frame slots. For interpreting 3D structures, the principle of angular invariance in rigid motion is used. Common line configurations, such as parallel and perpendicular lines, are dealt with by special computational units, implemented in LISP. Knowledge about the orientation of one line can simplify the interpretation of other lines in the same image. Experimentally, lines have been successfully interpreted by such a system.

Porrill [132] described a statistical method of fusion of geometric information obtained from multiple sensors, or from the same sensor at different times, known as GEOMSTAT. This method assumes that the scene sensed does not change significantly between measurements, or is constant for each sensing modality.
used. Requirements for the system are given. One requirement for such a method is that it perform in a uniform manner, independently of a reference frame. It should treat geometrical measurements and constraints naturally. For example, if it is decided that two lines meet at a vertex, less accurate measurements should be corrected to agree with more accurate ones. This reduces error and ensures integrity of the wireframe models. It should be able to handle correlated measurement errors. Sensor calibration parameters are to be treated in the same way as other measurement primitives. The first requirement is met by the appropriate choice of geometrical measurement primitives and by parameterizing their errors with respect to implicitly defined reference frames. For example, a point measurement is represented by a vector \( p_0 \) and deviations from it by means of \( p_0 + ap_1 + bp_2 + cp_3 \). The perturbation vector is \( x = (a, b, c)' \). The complete point measurement is described by the structure \((p_0, p_1, p_2, p_3, \bar{x}, S)\), where \( \bar{x} \) and \( S \) are the mean and covariance of \( x \). The other requirements are met by the use of generalized Gauss–Markov estimation for sensor fusion. The implementation makes use of a recursive Kalman filter. The main difference between this method and other methods in the literature is that this method can deal with constrained problems and that there is error correlation between all model elements and all sensor calibration parameters. Constraint imposition and measurement acquisition are both treated in the same manner.

Richardson and Marsh [133] described a multisensor fusion methodology to combine statistically independent information. Assume that a vector \( x \) repre-
resents the actual state of the system of interest. A vector $y$, which need not be of the same dimension as $x$, represents the measured quantities, and can include information from one or several sensors. This method assumes that the experimental error is additive and statistically independent from the state $x$, and that the probability densities of both $x$ and $y$ are known. Bayesian decision theory is used to estimate $x$ from $y$. A loss function, $L(\bar{x}, x)$, is first defined representing the penalty incurred when the state is estimated to be $\bar{x}$ while it is actually $x$. This function is multiplied by the probability $P(x, y)$ and integrated with respect to $x$ and $y$ to estimate the risk, $R$. Minimizing this functional with respect to $\bar{x}$ while holding $y$ fixed yields the optimal estimator, $\bar{x}_{opt}$. A proof, which shows that the use of additional sensors can never worsen the estimate of the state vector $x$, is given. It is also shown also that the use of additional sensors will usually improve the estimate of $x$ if an optimal state estimator is used. The main issue is how to decompose the estimation process into stand-alone modules. For this modularization, it is required that sensor measurements ($y_1$ and $y_2$) and additive error terms be statistically independent. Any common factor contributing to both errors can be removed by including it in the state vector. If no further assumptions are made, the processing required before the fusion is the derivation of $P(x|y_1)$ and $P(x|y_2)$, or $P(y_1|x)$ and $P(y_2|x)$. Other cases involving additional assumptions are considered also; for example the case where $P(x|y_1)$ and $P(x|y_2)$ are Gaussian and linear, in which the a posteriori means and covariances of $x$ with $y_1$, and of $x$ with $y_2$, suffice to determine the
required \( P(x|y_1) \) and \( P(x|y_2) \). This fusion method was successfully applied to data obtained by an acoustic pulse-echo sensor and an optical sensor. The state vector \( x \) was simply the elevation of a tetrahedral object as a function of the position in a base plane. The optical sensor provided complete information about the shape of the elevation function, but not about the absolute elevation, while the acoustic sensor provided complete information about the absolute elevation, but incomplete information about the shape.

Shekhar et al. [134] presented a method for the estimation of the position and orientation parameters of an object. The estimate of the orientation of an object is a nonlinear function of the measurements used to determine the estimate. The greater the distance between two points, the more accurate the orientation of the line they form. Orientation errors are a function of the distance between the points. The example of estimating the orientation of a planar object measured at five points is given. The actual locations of each of these points are known to exist within a cubic volume surrounding the measured point, rather than exactly at the measured point itself. Pairs of these measured points are combined to form vectors. The farthest two points are first combined, then the next farthest, etc... Vectors are then removed from the top of this list until all measured points are accounted for. This set of vectors is known as the set of best features. When these are used to estimate the orientation of the object’s surface in the least-squares sense, the least possible maximum expected error is obtained. If \( n \) is the number of points or object vertices used, there are \( n!/2 \) possible vectors, but
only \( n - 1 \) are necessary to account for all measurements. Thus, the set of best features consists of \( n - 1 \) vectors. The estimation of the object's position and orientation is found using quaternions to represent the transformation between the actual vectors as they exist on the object and the measured vectors. A weighting function is then found and the estimate determined in the least-squares sense. Bounds on the error are also found using this technique. This method was successfully implemented on a two-fingered, two degree-of-freedom robot, equipped with a tactile array and two sensors which output the centroid of the applied force and its total magnitude.

Stansfield [135] has described a perceptual system for the task of one-fingered robotic exploration of a single unmodeled object. The focus is on active touch and on the integration of touch and vision. Based on Fodor's perceptual model, this fusion scheme is hierarchical and modular. The system contains two subsystems: a vision system loosely based on Marr's vision model, and a haptic (active touch) system based on Lederman and Klatzky's work. Each module is highly specialized. It accepts inputs only from certain lower-level modules and sends its outputs only to certain higher-level modules. Some modules deal with visual data only, others with touch data only. Near the top of the hierarchy, a third type of module integrates the two sensing modalities. Initially, vision modules yield object location, 2- and 3-dimensional edges, and regions. Then, active tactile exploration of the object commences, from four different approach planes: top, front, left, and right. Depending on whether the type
of contact sensed is a point, an edge, or an extended contact surface, various modules invoke exploratory procedures (EP's) to extract additional properties from the object. Previously gathered tactile and visual information are both used to guide these EP's. The following tactile primitives are extracted: elasticity, compliance, surface-normal position, and roughness, as well as features such as points, edges, and contact areas. Intermediate-level tactile features extracted include edges, corners, and surface shapes. High-level tactile features extracted include contours, surface patches, and size and shape of components. Tactile and visual information is used to fill slots in frames which describe the object's components, features, and their interrelationships. This is known as object apprehension, as opposed to object recognition, which would go beyond what is described here to include attaching a label to the object or some of its components.
CHAPTER 4

Sensor and Data Modeling

In this chapter, a closer look is taken at the sensing process and the fusion process. The desirable characteristics of a fusion methodology are described. In a later chapter, various techniques will be examined to see if they meet these criteria.

4.1 Sensor Modeling

A sensor can be seen as a device which gives an assessment of the probability of the existence of a certain feature at the particular location in space at which the measurement was made (or at a particular location in an image). Also, the sensor can be seen as a device which assigns a membership value in a fuzzy set to each sensed data point.

4.2 Data Modeling

In the sensing process, raw data is obtained, from which can be inferred the location and strength of certain object features. This description can be in the form of a map, or image, of the object. This strength is usually scaled from 0 to
1, although other scalings are possible, and can be interpreted probabilistically: the greater the strength, the greater the probability of feature existence. These probabilities can then be fused using Bayesian or other methods. In this thesis, when interpreted probabilistically, 0 is taken to represent certainty of feature absence, 1 to represent certainty of feature presence, and 0.5 to represent total uncertainty, although other interpretations are possible. Since 0.5 is commonly taken to represent certainty that feature absence and feature presence are both equally likely, rather than a complete lack of knowledge about feature existence, one could argue that this does not represent total uncertainty about the existence of a feature. To have total uncertainty, the probability value must be unknown, rather than 0.5. This is true. Total uncertainty would require that it not be known whether feature existence and absence were equally likely. On the other hand, however, one could argue that a probability of 0.5 does represent uncertainty because the greatest inaccuracy in predicting feature existence occurs when one knows only that the probability of existence is 0.5. For example, when flipping a coin a large number of times, the result is not totally uncertain— it is known that approximately the same number of heads and tails will result. When flipping a coin only once, however, neither outcome (heads nor tails) is more certain, and thus the outcome can be described as unpredictable or uncertain. The probability value of 0.5 makes the existence of a feature more difficult to accurately predict than any other value, and for this reason is here defined as the most uncertain value, although admittedly in a sense, total uncertainty

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does not exist (since the probability value is not unknown). This problem can be avoided by the fuzzy set interpretation.

When interpreted as a fuzzy set membership value, 0 represents exclusion from the set of features, 1 represents membership in the set, while intermediate values represent partial membership in the set. As an example, consider the set of *young* individuals. An infant of age 10 months might definitely be a member of this set, with membership value of 1, while a 35 year old individual might be considered as having partial membership. Another example might be the set of mountains. Mount Everest would obviously have a membership value of 1, while an ant hill would have a value of 0, but some large, high hills may be considered to have partial membership in this set. In this thesis, a maximum pixel value (255) in edge maps indicates a membership value of 1 in the set of edges, while a minimum pixel value (0) indicates a membership value of 0. Intermediate pixel values indicate partial set membership.

The difference between the probabilistic and fuzzy interpretations is illustrated by the following example. A person whose age is exactly known may be given a fuzzy membership function value of 0.7 for being a *senior adult*. Another person, whose age is not known at all, may be given the probability of 0.7 for being a senior adult, based on the fact that 70% of the population in his neighborhood has been categorized as senior adults on the basis of the most recent census data. The first person is definitely not a teen-ager nor an infant, while the second person may possibly be one. To further illustrate the differ-

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ence, note that 0.5 as a fuzzy value means that the feature definitely does exist, to about one-half its maximum strength; whereas this could not be said if the same value were interpreted probabilistically, since the existence of the feature is totally uncertain, or unknown. The question of whether the feature's existence is dichotomatcal (the feature must either exist or not exist), or whether it can be said to exist to varying degrees, should determine which interpretation given to the sensed data. In any case, a two-dimensional mathematical function of either the relative probability of a feature's existence, showing the most likely locations in the image to find the feature, or of the feature's fuzzy membership value, showing the degree to which the feature exists at each location in the image, can be obtained. Often, the same data can be interpreted in both ways, with the preferred interpretation depending on the nature of the data, the features extracted, and the application.

4.2.1 An Example

In this example, the data are acquired by sensing the component of reaction force at contact in the x-direction. The greater the force measured, the greater the image intensity. The result is an image in which a rough outline of the shape of the object sensed is evident. An edge-detector algorithm is then applied to this image, resulting in the two-dimensional edge function illustrated by Fig. 1.2. By scaling these intensities from 0 to 1, they can be interpreted as representing the probability of that point's lying on an edge, or the value of
a fuzzy membership function, with 1 indicating the point is a member of an edge, 0 indicating the point is not a member of an edge, and intervening values representing intermediate degrees of edge-membership.

Data are also obtained by sensing the component of reaction force at contact in the y-direction. This also results in an image in which a rough outline of the shape of the object sensed can be seen. An edge-detector algorithm is also applied to this image, resulting in the two-dimensional edge function illustrated by Fig. 1.4.

4.3 Problem Description

The two edge map images described above are now to be fused. The fusion problem is, starting with two fuzzy membership functions, or two descriptions of the probability of existence of certain features at certain locations, obtained by two different sensors (or the same sensor at different times), to generate a fuzzy membership function, or object (scene) description of the probability of certain features, $\mu$ containing information from both sensors, as well as possibly information not obtainable from either sensor alone (or from either set of data alone, or from either input fuzzy membership function alone, or from either input probability description alone). This results in a final output edge map, shown in Fig. 1.5. The method of fusion can be one of many techniques. Several such techniques based on evidential reasoning are described in the next chapter.
The question of how to choose an appropriate fusion technique is addressed in the following section.

4.4 What is Desired in Fusion Methodology?

When one decides to fuse data, one of the first questions to be answered is, which fusion technique should be used? In answering this question, whether it be in the design of a fusion technique, or in the choice of an existing technique, one should be guided by the desired properties or characteristics of the function. In this section, some basic practical requirements of sensor data fusion are considered.

When a piece of information is acquired from a sensor about a scene, this information may be insufficient — more knowledge about that scene may be required. However, this additional information may or may not be obtainable by the sensor which obtained the original data, perhaps, for example, due to the original sensor being a range sensor and the desired data being some numerals painted in black on a white background on a flat surface. In this case, a second sensor of a type (vision) different than the first is required.

Two such sensors are complementary, in that each provides information not provided by the other [5]. There are other examples of such sensor complementarity. Infrared sensors are good at detecting small area objects or features, such as points or edges, while sonar is typically poor at such operations. However,
sonar is better than infrared sensing for detecting depth. Also, sensors have operating limits. Sonar's usefulness is limited to about five meters. Stereo vision's range is also limited by the triangulation geometry required.

It should be noted that when a second sensor is utilized to provide information additional to that provided by a sensor, the new information provided by the second cannot be verified by the first sensor. The reason for using the second sensor to detect this information in the first place, was that the first sensor did not provide this information. Therefore, in this situation, the first sensor cannot verify the accuracy of the additional information provided by the second sensor.

However, in some cases, such verification of data obtained by a sensor may be desired before accepting it, such as when it is known there is a significant chance of error, or when there is a large random noise component in the sensor measurements.

From the above discussion, it is clear that two principles exist to guide the fusion process [136,137,138,139,140]. When an additional sensor is required to provide information not provided by a sensor, or of a type that the first sensor cannot detect, Knowledge Addition is needed. When a second sensor is required to verify or confirm information provided by a sensor, Knowledge Confirmation is needed. Knowledge Confirmation can also be referred to as Knowledge Verification, Knowledge Consistency, or Belief Enhancement/Withdrawal, since if the belief of one sensor is not verified, it is withdrawn, thus resulting in an output
whose belief is enhanced by confirmation.

It is clear that these two principles are at conflict with each other. If one sensor reports the existence of a certain feature while a second does not, the first principle would cause this feature to be included in the output, while the second principle would cause it to be eliminated from the output. These principles represent opposite extremes; obviously, they cannot be satisfied simultaneously. The principle of Knowledge Addition yields more information than either input alone, yet this information is not completely confirmed by both sensors, and is thus somewhat questionable. The principle of Knowledge Confirmation yields less information, yet this information is completely confirmed and verified, and thus more reliable. The choice of which of these principles should be the guiding rationale in sensor fusion should be determined by the intended application.

At this point, consider a hypothetical example, and see which principle would be appropriate to guide the fusion process. Consider the case of two different sensors investigating the same object. One sensor may be a vision sensor, and the other a tactile array. Holes in the surface of an object can be detected by both vision (given appropriate lighting) and the tactile array.

It may be desirable to confirm features (holes) apparent in the vision image to ensure that they are not artifacts produced by shadowing, etc. This calls for the principle of Knowledge Confirmation. However, it may also be desired to include information about the exterior boundary of the object. While the tactile sensor can detect the extent of the boundary of the upper surface of the hypothetical
object, the exterior boundary may not be apparent to the vision sensor because of rounded edges on the object, the coloring of the sides of the object, etc. This means that knowledge provided by the tactile sensor is desired to be added to that provided by the vision sensor. Thus, the principle of Knowledge Addition is called for.

In many applications, as in the above example, a sensor will be called upon both to provide additional information and to verify existing information. However, Knowledge Confirmation implies that no additional knowledge is added, and that Knowledge Addition implies that additional knowledge (not supplied by another sensor) is not confirmed. How can these conflicting requirements be met? Is it possible to simultaneously utilize both principles for such cases? or, can a compromise—method that gives some of the advantages, though perhaps not the full advantages, provided by each method, be utilized? Some form of compromise is desired between these two conflicting principles so that they can coalesce into both an elegant fusion theory and a practical tool to address problems in robotic multi-sensing. This issue will be dealt with more fully in later chapters.

In addition to these two desired (though conflicting) characteristics of fusion functions, other generally desired characteristics are listed below:

1. When both inputs are 1, indicating total certainty about a feature's existence, or a maximum value of the membership function, there is no reason to conclude that the feature does not exist, or to doubt the veracity of ei-
ther sensor's assessment. On the contrary, there is every reason to believe in the feature's existence. Therefore, the output should be 1 [141].

2. Conversely, when both assessments are 0, indicating certainty of the feature's absence as opposed to its presence, it is desired that the output be 0, for similar reasons [141].

3. The presence or absence of a given feature should not depend upon the order of the use of the sensors; therefore, another desirable property of a fusion function is symmetry, or commutativity, and associativity; that is, the order of the inputs should have no effect on the output [141,142].

4. Now consider the domain from which the input values are taken. In general, and ideally, this would be an interval of real numbers. This being the case, why should one point (input) in this domain have a vastly different effect upon the output than its nearby neighbor? Obviously, this is not desired [142]. Therefore, the fusion function should be continuous. Also, continuous functions are, in general, easier to deal with mathematically.

5. Why should there be an abrupt change from one point to another in the input domain, in the slope or derivative of the output? This is not desired, either. Thus, a differentiable function is preferred [142].

6. Stronger evidence for belief (a higher input value) should obviously result in a larger belief value (a higher output value). Thus, monotonicity is
desired; the fusion function should output a higher value when both inputs are high than it does when both inputs are low; i.e., if \( x \geq p \) and \( y \geq q \), then \( f(x, y) \geq f(p, q) \) [141,142].

7. Irrelevant evidence should not affect the output. This requires the existence of an identity element, \( i \), such that \( f(i, x) = x \), for all \( x \). This will usually (and in this thesis) be represented by the input value of 0.5, which indicates maximum uncertainty. Values less than \( i \) are said to be “negative”, and should have the effect of decreasing the degree of belief in the feature’s existence, while “positive” values are greater than \( i \) and should increase the degree of belief in the feature’s existence [142].

8. When both inputs are totally uncertain (0.5), there is no basis for the fusion function to conclude anything; therefore, in such a case, the output is required to be totally uncertain (0.5) [141].

9. Also, when one sensor exhibits total belief in a feature’s absence while another sensor exhibits total belief in that feature’s presence, it is desired that the output be somewhere between these extremes of total certainty about the feature’s existence; otherwise, the totally certain output would ignore one of the inputs, which would be just as certain that the output is wrong as the other input is certain that it is right. In general, the fusion function is desired to be more flexible than this in its range of applications [141].
10. It is desirable that the set of axioms satisfied by an operator be as unlim-
  iting as possible [122].

11. The operators must appropriately exemplify the behavior of the real sys-
  tem they model. This can normally be assured only by empirical test-
  ing [122].

12. It would be more efficient to only use a small number of operators to
  model many situations. Thus, a desired characteristic of an operator is
  adaptability. The operator, however, is usually dependent upon its context
  or application, and limited in its adaptability. One method of increasing
  the adaptability of operators is by the use of parameterization, which is
  used in the analytic fusion method discussed later in this thesis [122].

13. An obviously desirable attribute is that the computational effort required
  to implement the operator be minimal [122].

14. Often the output of the fusion function can be changed by altering the
  value of one of the inputs. It is desirable that this output change be
  preventable by changing the other input, or that the function be compen-
  satory [122].

15. In general, the range of resulting membership should be as large as possi-
  ble [122].

16. While continuity and differentiability guarantee that small changes in the
input result in small changes in the output, the magnitude of the resultant output change may vary. For example, if the input 0.4 is increased by 0.001, the output may increase by the amount 0.1. However, if a different input, say 0.88, is increased by the same amount, the output may vary by 0.7, instead of by 0.1! The global robustness of a (differentiable) fusion function is defined as $R = \max |\partial f(x, y)/\partial a|$, and indicates the maximum ratio of change in output to change in input [142]. Either too small or too large a value for $R$ is undesirable.
CHAPTER 5

Classical Combination Functions

This chapter focuses on fusion or inference techniques based on evidential reasoning methods.

The probabilities or membership functions considered here indicate a belief about the existence of a token or object feature (edge, corner, surface, etc.). They represent data sensed by two sensors, and are also the two inputs to a fusion function. To avoid inconsistency between the sensed data, the two inputs will always satisfy the following:

$$0 < x < 1 \text{ and } 0 < y < 1,$$

In invoking very low probability or membership function values ($0^+$), let $x$ or $y$ be equal to 0. Similarly, in invoking very high probability or membership function values ($1^-$), let $x$ or $y$ be equal to 1. In other words, in the case of total certainty of the absence of a feature, the sensor would output 0; in the case of total certainty of the existence of a feature, the sensor would output 1. It is logical to conclude that in the case of total uncertainty about the existence (or nonexistence) of a feature, the sensor should output 0.5. This means that the
sensor has not formed a belief about the given event; i.e., its assessment neither supports nor denies the existence (or nonexistence) of a feature.

The functions discussed in this chapter are shown in Table 5.1 [126,142]. These functions are used to combine one-dimensional Gaussian functions. These functions are also plotted as two-dimensional functions of the two input assessments (probabilities or membership functions) from two sensors. One basis for comparing these results is to consider the relative amounts of Knowledge Addition and Knowledge Verification involved in the fusion process. This can be done by considering the volume (or area) under the surface (or curve) defined by the output. If fusion is accomplished utilizing only Knowledge Addition, each taxel value in the fused result will have a value greater than or equal to the corresponding value in either input image. On the other hand, if the principle of Belief Enhancement/Withdrawal is utilized solely in the fusion process, the value of each taxel in the result will then be less than or equal to that of the corresponding taxels in the input images. If one considers the resultant fused image to be a function of the two independent spatial variables x and y, then the volume beneath the two-dimensional surface representing this function will be larger if the principle of Knowledge Addition is used, and smaller if the principle of Belief Enhancement/Withdrawal is used.

Thus, this volume can give some indication of the degree of Knowledge Addition versus Belief Enhancement/Withdrawal involved in the fusion process. For the same two input images, the greater the amount of Knowledge Addition
Table 5.1: Evidential reasoning based functions

<table>
<thead>
<tr>
<th>Function</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1(x,y)$</td>
<td>$\min(x,y)$</td>
</tr>
<tr>
<td>$f_2(x,y)$</td>
<td>$xy$</td>
</tr>
<tr>
<td>$f_3(x,y)$</td>
<td>$\max(x+y-1,0)$</td>
</tr>
<tr>
<td>$f_4(x,y)$</td>
<td>$\max(x,y)$</td>
</tr>
<tr>
<td>$f_5(x,y)$</td>
<td>$x+y-xy$</td>
</tr>
<tr>
<td>$f_6(x,y)$</td>
<td>$\min(x+y,1)$</td>
</tr>
<tr>
<td>$f_7(x,y)$</td>
<td>$\frac{\min(x,y)}{1-</td>
</tr>
<tr>
<td>$f_8(x,y)$</td>
<td>$\frac{\max(x,y)}{1+</td>
</tr>
<tr>
<td>$f_9(x,y)$</td>
<td>$\frac{xy}{1-x-y+2xy}$</td>
</tr>
<tr>
<td>$f_{10}(x,y)$</td>
<td>$\frac{x+y-xy}{1+x+y-2xy}$</td>
</tr>
<tr>
<td>$f_{11}(x,y)$</td>
<td>$\sqrt{xy}$</td>
</tr>
<tr>
<td>$f_{12}(x,y)$</td>
<td>$1 - \sqrt{(1-x)(1-y)}$</td>
</tr>
<tr>
<td>$f_{13}(x,y)$</td>
<td>$\text{median}(x,y, .5)$</td>
</tr>
<tr>
<td>$f_{14}(x,y)$</td>
<td>$(x+y)/2$</td>
</tr>
<tr>
<td>$f_{15}(x,y)$</td>
<td>$\frac{\min(x,y)}{\max(\min(x,y), \min(1-x,1-y))}$</td>
</tr>
<tr>
<td>$f_{16}(x,y)$</td>
<td>$\frac{x+y}{1+xy}$</td>
</tr>
</tbody>
</table>
involved in their fusion, the greater the volume underneath the two-dimensional surface represented by the fused result. The greater the amount of Knowledge Verification, the smaller the volume underneath this surface. The input images were identical for all these fusion methods. This number, representing the volume underneath the surface of the fused result, will vary not only as a function of the relative amounts of Knowledge Addition versus Knowledge Corroboration, but also as a function of the inputs. However, a measure of Knowledge Addition vs. Knowledge Verification can be defined which is independent of input images, and is based on the fusion function alone. This can be estimated by computing the area (or volume) under the curve of this fusion function as \( x \) and \( y \) are increased linearly from 0 to 1. The output is unique for each fusion method. The volume underneath the surface generated in such a manner is a characteristic of the corresponding fusion function, indicating the degree of Knowledge Addition versus Belief Enhancement/Withdrawal; the greater this volume, the more the function favors the principle of Knowledge Addition, and the smaller this volume, the more the function favors Belief Enhancement/Withdrawal. This volume is herein referred to as the *ideal volume*. The *actual volume* is the volume underneath the surface of the fused result, which depends also upon the inputs. For each of the 16 fusion methods described in this chapter, the ideal and actual volumes have been measured, scaled, and tabulated in Table 5.2.

Some statements can be made which apply to all the functions discussed below. One is that they all are nondecreasing in the range \([0, 1]\) for both inputs.
Table 5.2: Evidential reasoning based fusion functions ranked according to increasing Knowledge Addition

<table>
<thead>
<tr>
<th>Function</th>
<th>Actual Volume</th>
<th>Ideal Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_3 : \max(x + y - 1, 0) )</td>
<td>0.080</td>
<td>0.200</td>
</tr>
<tr>
<td>( f_2 : x \times y )</td>
<td>0.180</td>
<td>0.300</td>
</tr>
<tr>
<td>( f_7 : \frac{\min(x,y)}{1-</td>
<td>x-y</td>
<td>} )</td>
</tr>
<tr>
<td>( f_9 : \frac{xy}{1-x-y+2xy} )</td>
<td>0.200</td>
<td>0.601</td>
</tr>
<tr>
<td>( f_1 : \min(x, y) )</td>
<td>0.213</td>
<td>0.400</td>
</tr>
<tr>
<td>( f_{11} : \sqrt{xy} )</td>
<td>0.253</td>
<td>0.534</td>
</tr>
<tr>
<td>( f_{15} : \frac{\min(x,y)}{\max(\min(x,y),\min(1-x,1-y))} )</td>
<td>0.320</td>
<td>0.827</td>
</tr>
<tr>
<td>( f_{12} : 1 - \sqrt{(1-x)(1-y)} )</td>
<td>0.533</td>
<td>0.669</td>
</tr>
<tr>
<td>( f_{14} : (x + y)/2 )</td>
<td>0.640</td>
<td>0.600</td>
</tr>
<tr>
<td>( f_4 : \max(x, y) )</td>
<td>0.793</td>
<td>0.801</td>
</tr>
<tr>
<td>( f_8 : \frac{\max(x,y)}{1+</td>
<td>x-y</td>
<td>} )</td>
</tr>
<tr>
<td>( f_5 : x + y - xy )</td>
<td>0.847</td>
<td>0.900</td>
</tr>
<tr>
<td>( f_{10} : \frac{x+y-xy}{1+x+y-2xy} )</td>
<td>0.853</td>
<td>0.601</td>
</tr>
<tr>
<td>( f_{16} : \frac{x+y}{1+xy} )</td>
<td>0.860</td>
<td>0.928</td>
</tr>
<tr>
<td>( f_6 : \min(x + y, 1) )</td>
<td>0.907</td>
<td>1.000</td>
</tr>
<tr>
<td>( f_{13} : \text{median } (x, y, 0.5) )</td>
<td>1.000</td>
<td>0.601</td>
</tr>
</tbody>
</table>
(All characteristics of these functions refer to this domain of definition for the functions.) Also, they are all symmetrical, in that \( f(x, y) = f(y, x) \). All yield an output of 1 for inputs \((1, 1)\), and an output of 0 for inputs of \((0, 0)\). The computational effort involved in evaluating these functions is limited, since they are for the most part very simple functions (e.g., \( \text{min}(x, y) \)).

Only one, \( f_9 \), has 0.5 as an identity element. Some functions, \( f_4, f_5, f_6, \) and \( f_{16} \), have 0 as their identity element. The identity element for \( f_1, f_2, \) and \( f_3 \) is 1. Those functions with 0 as their identity element have the undesirable characteristic that the belief of a feature's existence due to one input can never be decreased by the other input, even if that other input is certain the feature does not exist (0). Likewise, those functions with 1 as their identity element can never have the output belief increased, even by a second input of 1. Those functions without identity elements have no means of handling the input of irrelevant information, which should be ignored. Only one function, \( f_9 \), has the desired abilities to represent irrelevant input as well as to both increase and decrease the effect of one input by means of a second input.

Most of these functions are continuous. Those not continuous (\( f_7 \) and \( f_9 \)) have discontinuities only at the points \((0, 1)\) and \((1, 0)\). Less than half are differentiable at all points in their domain of definition. In the discussion below, the symbol \( \Delta x \) indicates a change in an input variable to a fusion function. The change is always expressed as \( x + \Delta x \), where \( \Delta x \) may be positive or negative. These functions are next considered individually.
The various functions considered herein are illustrated by an example: the case of two Knowledge Sources (e.g., sensor – hereafter referred to as KS) reporting separately about the probability of failure of an electronic module within a time period of one year.

1. $f_1(x, y) = \min(x, y)$. [Fig. 5.1.] The first function to be considered, $f_1$, is simply the minimum of the two inputs. This function is continuous everywhere, and differentiable everywhere except on the line $x = y$. Taking the minimum serves to eliminate erroneous, spurious information or noise, which would usually not be present in both sensor assessments. However, another type of error, that of eliminating information about existing features sensed by only one sensor, is exacerbated by this method. For example, if one KS assigns a probability of failure of the electronic module within the next year of 0.5 while the second KS assigns a probability of failure of 1, $f_1$ would ignore the KS which is certain of the module’s failure, and rely solely on the uncertain source of knowledge. This behavior results in an undesirable, significant loss of information. Therefore, this method favors the principle of Knowledge Verification, mentioned in Chapter 4. This can be seen in Fig. 5.1. The continuity of the outer circumference on the left side of Fig. 5.1 (d) is not present in the output, Fig. 5.1 (f). Also, the continuity of the top portion of the outer circumference shown in Fig. 5.1 (c) is not present in the output. Inputs of 0 and 1 result in an output of 0, which was previously mentioned as undesirable. Yet when both inputs are 0.5, the output is 0.5. A change of one input variable ($\Delta x$) results in either a decrease or no
Figure 5.1: $f_1(x, y) = \min(x, y)$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
change in the output and cannot be compensated for by changing the other input. Other changes can be compensated for, however, by \( \Delta y = \min(x, y) - y \).

2. \( f_2(x, y) = xy \). [Fig. 5.2.] The function \( f_2 \) performs a multiplicative product between the two input probabilities or membership function values. This function is continuous and differentiable everywhere, but when one sensor is absolutely certain of a feature's existence, and another sensor is absolutely certain of its absence, or the inputs are 0 and 1, the output is 0. The most reasonable output would be 0.5. Here, the sensor which is absolutely certain of the feature's existence is ignored. Also, when both inputs are equally, totally uncertain (0.5, 0.5), the output is not 0.5, as desired, but 0.25, indicating a bias toward belief in the feature's absence. As the previous function, this one also emphasizes the principle of Knowledge Confirmation, as can be seen in the breaks of the outer circumference in the output image [Fig. 5.2 (f)]. This function, however, not only supplies no additional information to one sensor's output, but outputs less information than provided by either sensor. For example, if both sensors measure 0.8 as the probability of feature existence, this function would output 0.64, a smaller value than either sensor provided. This function is compensatory only for any change in an input (\( \Delta x \)) for which the following holds:

\[
\Delta x \geq xy - x
\]
Figure 5.2: $f_2(x, y) = xy$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
where $x, y$ are the two inputs. Then,

$$
\Delta y = -\Delta xy/(x + \Delta x)
$$

3. $f_3(x, y) = \max(x + y - 1, 0)$. [Fig. 5.3.] The function $f_3$ is everywhere continuous and differentiable everywhere except along the line $x + y = 1$. When the inputs are $(0, 1)$ or $(1, 0)$, the output is not the desired 0.5, but 0. In this case, $f_3$ systematically ignores one of the assessments. This is always true when one input is 0. Consider, for instance, the example mentioned earlier. If a KS supports failure with a belief of 0, regardless of how high the belief of the second KS may be, this fusion function always outputs a belief of 0. This is true even if the second KS is certain of module failure (belief equal to 1). If both KS's are not sure about the module’s failure (belief of 0.5 from both), $f_3$ wrongly concludes that the module definitely will not fail (output belief of 0). If a KS supports failure with a belief of 1, and the second KS is completely uncertain (belief of 0.5), the final assessment is uncertain. Indeed, when one input is 1, this input is always disregarded: the output will simply be the value of the second input! Under this scheme of reasoning, a certain assessment (belief of 1) is systematically discarded. This function, as the two previous ones, favors Knowledge Confirmation. However, the output contains less information than either input. Even some information present in both inputs is eliminated by this
function. For example, the small center hub is missing from the output, though present in both inputs. Also, the multiple arcs present in a large portion of the outer circumference have been largely eliminated. When both inputs are 0.5 or less, the output is 0. Therefore, this function may be useful where strong certainty of feature presence is required, but it overemphasizes certainty of feature absence. The change in an input variable can be compensated for only when

\[ y - 1 \leq \Delta x \leq y, \]

where \( x, y \) are the two inputs, by

\[ \Delta y = -\Delta x. \]

4. \( f_4(x, y) = \max(x, y) \). [Fig. 5.4] The function \( f_4 \), the maximum of the two sensor assessments, is obviously biased in favor of belief in the existence of a feature. This can be seen by comparing Fig. 5.4 (f) and Fig. 5.3 (f), the outputs of \( f_4 \) and \( f_3 \). \( F_4 \) emphasizes Knowledge Addition. Shortcomings of such a function are easily deduced from those of \( f_3 \) or \( f_1 \), which are also biased, but in the opposite direction. Like \( f_1 \), this function is everywhere continuous, but not differentiable on the line \( x = y \). An assessment of 0, which is absolutely certain the feature is absent, is always ignored in favor of the other assessment, even
Figure 5.4: $f_s(x, y) = \max(x, y)$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
if the other is absolutely uncertain (belief of 0.5). Obviously, inputs of (0, 1) result in an output of 1 instead of the desired 0.5. The desired output of 0.5 is obtained, however, for the inputs (0.5, 0.5). This function is compensatory only when $\Delta x$ increases the output value, by

$$\Delta y = \max(x, y) - y.$$ 

5. $f_5(x, y) = x + y - xy$. [Fig. 5.5.] The function $f_5$ is continuous and differentiable everywhere. However, it always ignores an assessment that carries a belief of 0. In other words, if a KS supports total disbelief (belief of 0), the outcome is equal to that of the second KS. Here, a maximally certain assessment is systematically ignored – certainly undesirable behavior, in general. By the same token, an assessment of total belief (1) by a KS is always selected over any assessment by a second KS, even over an equally certain assessment of 0. Again, an assessment is systematically ignored. When both assessments are completely uncertain of module failure (belief of 0.5), the output moves toward certainty of module failure (0.75). Such behavior is obviously undesirable because a no-opinion assessment (0.5) seems more appropriate. Since the outcome is always greater than either input, this function is biased in favor of Knowledge Addition.

It is compensatory only where

$$\Delta x \leq y - xy$$

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Figure 5.5: $f_{5}(x, y) = x + y - xy$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
To compensate for \( \Delta x \),

\[
\Delta y = \frac{\Delta x(y - 1)}{(1 - x - \Delta x)}
\]

6. \( f_6(x, y) = \min(x + y, 1) \). [Fig. 5.6.] This function is continuous everywhere, and differentiable everywhere except along the line \( x + y = 1 \). This function suffers from several drawbacks. One is that if the input from one KS is 1, the other KS is ignored, regardless of its assessment – even if that assessment is 0. Often, it is not prudent on the part of a knowledge fusor to be unjustifiably biased toward one KS or another. Another disadvantage of this rule of combination is the assignment of absolute certainty (belief of 1) in the face of two uncertain assessments (belief of 0.5). Similarly, two inputs, both favoring belief in the nonoccurrence of an event (0.25 of belief, for instance), can result in total uncertainty about the event (belief of 0.5). The output of this function is always greater than or equal to the largest input; thus, this method is biased toward belief in event occurrence, or the principle of Knowledge Addition. If both sensors give a feature a fuzzy membership value of 0.8, it seems reasonable to conclude that 0.8 is the best estimate obtainable with this information. However, this fusion method outputs a value of 1 for these inputs. From the probabilistic viewpoint, if two sensors agree that the probability of feature exis-
Figure 5.6: $f_6(x, y) = \min(x + y, 1)$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
tence is 0.8, this function concludes from that information that the probability of feature existence is 1, which seems an unjustifiable conclusion. The result of this bias toward feature existence can be seen in Fig. 5.6. Of the three outer rings in the output image, the center ring is brighter than it appears in either of the two inputs, as can be seen in Figs. 5.6 (c), (d), and (f). This is also true for other parts of the output image. This can also be seen by comparing Fig. 5.6 (b) with Fig. 5.3 (b). The area underneath the curves representing fusion results is greater when using \( f_6 \) than \( f_3 \). This function cannot be compensated when \( x + y < 1 \) and \( \Delta x > y \).

When compensation is possible, the compensating \( \Delta y = -\Delta x \).

7. \( f_7(x, y) = \min(x, y)/(1 - |x - y|) \). [Fig. 5.7.] This function is not continuous at the points \((0, 1)\) and \((1, 0)\). It is not differentiable at these points nor on the line \( x = y \). The function \( f_7 \) is biased toward whichever assessment is the more certain of the two. (Certainty here is loosely defined as the unsigned deviation from 0.5.) This is often highly desirable in a rule of combination. However, if either KS is absolutely certain (belief of either 0 or 1), the other KS is ignored. This is undesirable. When both KS's disagree with absolute certainty (one's belief is 0, the other is 1), this function cannot be evaluated. Unfortunately, this holds true even at the limit, which is not unique when approached from different directions. If one assessment is held constant at 1 and the other allowed to approach 0, the limit is 1; but when the other assessment
Figure 5.7: \( f_7(x, y) = \min(x, y)/(1 - |x - y|) \): (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
is varied, the limit is 0. However, this function does yield the desired output of 0.5 for inputs of (0.5, 0.5). This function is not compensatory.

8. \( f_8(x, y) = \max(x, y)/(1 + |x - y|) \). [Fig. 5.8.] This function is continuous everywhere, but not differentiable along the line \( x = y \). The function \( f_8 \) favors uncertainty, rather than a belief in the absence or presence of a feature. The result is always nearer the more uncertain assessment (the one which is closer to 0.5). For example, beliefs of 0.8 and 0.9 yield a final assessment of 0.82, while beliefs of 0.1 and 0.2 yield an assessment of 0.18. In both cases, the output is nearer the assessment which is more uncertain about the event’s occurrence. Uncertain assessments carry more weight than do more certain ones. Although this may not seem intuitively desirable, for certain applications in which extreme caution is desired, this function may be appropriate. However, other applications would require a different fusion function with a more assertive assessment from a rule of combination. This function does have the desirable attribute of yielding an output of 0.5 for inputs of (0, 1) and (1, 0). However, the inputs of (0.5, 0.5) result in an output of 0.33 instead of the desired 0.5, again indicating a bias toward caution. The effect of the inputs upon the relative amounts of Knowledge Addition and Knowledge Confirmation involved in their fusion can be seen in Figs. 5.7 (f) and 5.8 (f). Although both have the same ideal volume (see Table 5.2), their actual volumes are quite different. This function is not compensatory.

9. \( f_9(x, y) = xy/(1 - x - y + 2xy) \). [Fig. 5.9.] The function \( f_9 \) is both
Figure 5.8: $f_s(x, y) = \max(x, y)/(1 + |x - y|)$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
Figure 5.9: \( f_0(x, y) = xy/(1 - x - y + 2xy) \): (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
continuous and differentiable everywhere except at the points \((0, 1)\) and \((1, 0)\), where it is neither. This function, as opposed to \(f_8\), is less cautious and favors the more certain assessment. However, in some cases, the final assessment is more certain than either of the input beliefs, which is not always desirable. For example, two KS's both supporting an event with a belief of 0.1 yield a final assessment of 0.012. Similarly, two KS’s both supporting an event with a belief of 0.9 yield a final assessment of 0.988. This function is not defined for two absolutely certain, but opposite, inputs (beliefs of 0 and 1). Its behavior is similar to that of \(f_7\); both methods cannot handle disagreement when both input assessments are absolutely certain. However, when both inputs are 0.5, the output is the desired 0.5. This function is not compensatory.

10. \(f_{10}(x, y) = (x + y - xy)/(1+x+y-2xy)\). [Fig. 5.10.] This function is continuous and differentiable everywhere. It favors uncertainty. When presented with two beliefs, \(f_{10}\) yields an assessment which is always numerically closer to the more uncertain KS of the two. For example, if the belief inputs about the feature existence are both 0.1, this function yields an assessment of 0.16, which is more uncertain than either input. If both inputs are 0.9, \(f_{10}\) yields a final belief of 0.84, which is also more uncertain than either input. When presented with assessments which contradict each other about the feature’s existence or nonexistence, the output is also nearer the more uncertain of the two assessments. For example, a belief of 0.1 indicates a very strong belief in a feature’s absence, while a belief of 0.6 indicates a very weak belief in the feature’s presence. The
Figure 5.10: $f_{10}(x, y) = (x + y - xy)/(1 + x + y - 2xy)$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
outcome of $f_{10}$ for these inputs is 0.41, which is closer to the weak assessment than to the more certain one. This may be appropriate for some applications, but not for others. This indicates that in some aspects, these functions are somewhat inflexible. This function has the desirable quality that for inputs of $(0, 1)$ and $(1, 0)$, the output is 0.5. This is also true for inputs of $(0.5, 0.5)$. This function is not compensatory.

11. $f_{11}(x, y) = \sqrt{xy}$. [Fig. 5.11.] This function has no discontinuities, and only one point at which it is not differentiable: $(0, 0)$. It favors belief in an event's nonoccurrence, or in a feature's absence. This is not the same as favoring uncertainty, nor is it favoring certainty, for either of these would result in a bias with respect to the middle of the range $[0, 1]$, either toward or away from 0.5. $F_{11}$, however, is biased with respect to the minimum possible value, 0, in that the output is nearer the minimum input. This function ignores one input, regardless of its value, when the other input is 0. Inputs of 0.5 and 0.5 do result in the desired output of 0.5. This function is compensatory only when

$$\Delta x \geq xy - x$$

and,

$$\Delta y = \frac{-\Delta xy}{x + \Delta x}$$
Figure 5.11: \( f_{11}(x, y) = \sqrt{xy} \): (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
12. $f_{12}(x, y) = 1 - \sqrt{(1 - x)(1 - y)}$. [Fig. 5.12.] The function $f_{12}$ is continuous everywhere, and not differentiable at only one point: (1, 1). It differs from $f_{11}$ in that it is biased toward belief in feature existence. This is the influence of the principle of Knowledge Addition. For example, given two input assessments of 0.1 and 0.9, both equally certain (the same distance from 0.5), the output belief, 0.7, leans heavily toward the assessment in favor of feature existence. One input KS is always ignored, regardless of its certainty, when the other input is 1. Therefore, inputs of 1 and 0 result in the output of 1. Inputs of 0.5 and 0.5 yield the desired result of 0.5. This function is compensatory only when

$$\Delta x \leq y - xy$$

and,

$$\Delta y = \Delta x(y - 1)/(1 - x - \Delta x)$$

13. $f_{13}(x, y) = \text{median}(x, y, 0.5)$. [Fig. 5.13.] This is one of the simplest combination rules. Function $f_{13}$ responds with the median of a totally uncertain belief of 0.5 and the two inputs. It is continuous everywhere, but is not differentiable when $x = y > 0.5$ or when $x = y < 0.5$, or along the line $x = y$, except at the point (0.5,0.5). When the two KS's inputs disagree about a feature's
Figure 5.12: $f_{12}(x, y) = 1 - \sqrt{(1 - x)(1 - y)}$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
Figure 5.13: $f_{13}(x, y) = \text{median}(x, y, 0.5)$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
existence, the final outcome is always 0.5: total uncertainty. It would seem that if one KS has a belief of 1 that a feature exists, and another KS has a belief of 0.49 that the feature exists, the first KS should be given greater influence in the determination of the output, since it is absolutely certain about the feature's existence, while the other KS is almost totally uncertain, only weakly favoring a belief in the feature's nonexistence. Yet the output would be much nearer the uncertain KS's assessment than the certain one's. Also, when both assessments agree (i.e., they are both either greater than or less than 0.5), $f_{13}$ outputs the more uncertain (closer to 0.5) of the two assessments. The more sure assessment is ignored. This is the case even if one input is absolutely certain (belief of 1), while the other is very unsure (probability of 0.51). Here, a very uncertain assessment (0.51) is allowed to change an absolutely certain assessment (1) to one of almost complete uncertainty, which is generally undesirable. Although this may be appropriate for some applications, it may not be so for other applications. Thus, this function, as many others, is limited in applicability to only certain situations. This function does have the desirable characteristic that inputs of 1 and 0 result in an output of 0.5, while inputs of 0.5 and 0.5 also yield 0.5 as the output. Under certain conditions, a change of $\Delta x$ in one input ($x$) can be compensated for by a change $\Delta y$ in the other input ($y$). However, the $\Delta y$ required varies for different conditions. There are six different cases, as shown in Table 5.3.

14. $f_{14}(x, y) = \frac{(x + y)}{2}$. [Fig. 5.14.] The function $f_{14}$ is the simple
Table 5.3: Conditions under which $f_{13}$ may be compensated and required compensation $\Delta y$.

<table>
<thead>
<tr>
<th>Input Conditions</th>
<th>Initial Change $\Delta x$</th>
<th>Compensating Change $\Delta y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x \leq 1/2 \leq y$</td>
<td>$\Delta x \leq 1/2 - x$</td>
<td>$\Delta y \geq 1/2 - y$</td>
</tr>
<tr>
<td></td>
<td>$\Delta x \geq 1/2 - x$</td>
<td>$\Delta y \leq 1/2 - y$</td>
</tr>
<tr>
<td>$y \leq 1/2 \leq x$</td>
<td>$\Delta x \geq 1/2 - x$</td>
<td>$\Delta y \leq 1/2 - y$</td>
</tr>
<tr>
<td></td>
<td>$\Delta x \leq 1/2 - x$</td>
<td>$\Delta y \geq 1/2 - y$</td>
</tr>
<tr>
<td>$x \leq y &lt; 1/2$</td>
<td>$\Delta x \leq y - x$</td>
<td>$\Delta y = 0$</td>
</tr>
<tr>
<td></td>
<td>$\Delta x = y - x$</td>
<td>$\Delta y \leq 0$</td>
</tr>
<tr>
<td>$y \leq x &lt; 1/2$</td>
<td>$\Delta x \leq 0$</td>
<td>$\Delta y = x - y$</td>
</tr>
<tr>
<td></td>
<td>$\Delta x = 0$</td>
<td>$\Delta y \leq x - y$</td>
</tr>
<tr>
<td>$1/2 &lt; x \leq y$</td>
<td>$\Delta x = 0$</td>
<td>$\Delta y \geq x - y$</td>
</tr>
<tr>
<td></td>
<td>$\Delta x \geq 0$</td>
<td>$\Delta y = x - y$</td>
</tr>
<tr>
<td>$1/2 &lt; y \leq x$</td>
<td>$\Delta x = y - x$</td>
<td>$\Delta y \geq 0$</td>
</tr>
<tr>
<td></td>
<td>$\Delta x \geq y - x$</td>
<td>$\Delta y = 0$</td>
</tr>
</tbody>
</table>
Figure 5.14: $f_{14}(x, y) = (x + y)/2$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
arithmetic average of two inputs. It is everywhere both continuous and differentiable. It gives the same weight to totally uncertain assessments (belief of 0.5) as to totally certain ones (belief of 1 or 0). Inputs of 1 and 0 yield the output of 0.5, as do inputs of 0.5 and 0.5, as desired. A change of $\Delta x$ can be compensated for by a change $\Delta y = -\Delta x$ only when $(y - 1) \leq \Delta x \leq y$.

15. $f_{15}(x, y) = \min(x, y)/\max(\min(x, y), \min(1 - x, 1 - y))$. [Fig. 5.15.] The function $f_{15}$ has no discontinuities, but is not differentiable on the line $x = y$, and also is not differentiable when the following condition holds: $x + y = 1$. When either input is extreme (belief of 0 or 1), the other input is ignored. If the other input happens to be the opposite extreme value, $f_{15}$ is undefined and the limit does not exist. (When holding one input constant at 0 and letting the second input approach 1, the limit is 0. When the first input is varied, however, the limit is 1.) When both inputs are uncertain (0.5, 0.5), the output should be as uncertain as the inputs, but instead is absolutely certain, 1. It can be seen by examining Fig. 5.15 (e) that if input values are low (less than 0.5), this function favors Knowledge Confirmation, while it favors Knowledge Addition when input values are high. Here again is seen the dependency of the fusion technique upon the input values. This function is not compensatory.

16. $f_{16}(x, y) = (x + y)/(1 + xy)$. [Fig. 5.16.] This function is continuous and differentiable everywhere. However, when the inputs are 0 and 1, the output is 1, thus ignoring an absolutely certain input (0). The output is also biased toward belief in a feature’s existence (0.8) when the inputs are as uncertain as
Figure 5.15: $f_{15}(x, y) = \min(x, y)/\max(\min(x, y), \min(1 - x, 1 - y))$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
Figure 5.16: $f_{16}(x, y) = (x + y)/(1 + xy)$: (a) 1-d fusion inputs; (b) 1-d fusion results; (c), (d) 2-d fusion inputs; (e) 2-d ideal surface; (f) 2-d fused result.
possible (0.5 and 0.5). When the change

\[ \Delta x \leq y(1-x^2)/(y(x+y)+1-y^2) \]

this function can be compensated by

\[ \Delta y = \Delta x(1-y^2)/(x^2 + \Delta x(x+y) - 1). \]

Functions \( f_i \) through \( f_{16} \) are very general; their inputs can be any physical quantity that can be modeled by a probability or membership function. In this work, the inputs indicate the strength of a feature or token extracted from a contact (typically force) image.

Consider fusion function \( f_{13} \), the \text{median}(x, y, 0.5). When the criteria given in Table 5.3 for \( \Delta y \) are not met, the function is not compensatory. In practice, it will probably not be known which of the six cases listed in Table 5.3 holds, and thus whether or not the function is compensatory for certain inputs, until the two Knowledge Sources (KS) actually input their assessments to the fusion function. If a compensatory function is required, one must either assume \( f_{13} \) not to be compensatory, or test whether it is for each new set of inputs, thus slowing down the processing of the data. As can be seen above, different evidential reasoning based fusion functions have different criteria not only for compensation, but also for continuity and differentiability. This can require extra time for derivation of
these constraints and for the programming of tests for these constraints, unless one limits him/herself in flexibility by the use of only one function.

In the next chapter, a new analytical fusion method will be examined.
CHAPTER 6

Analytic Fusion of Multisensory Data

In this chapter, some basic practical requirements of sensor data fusion are first considered, and then an analytical fusion method satisfying these requirements is derived [139,137,138,136,140]. In later chapters, results of experiments utilizing this new analytical fusion method will be given and compared with the evidential reasoning based methods of the previous chapter.

6.1 Introduction

Two basic functions of, and reasons for, multi-sensor fusion were considered in Chapter 4. They were labeled as the principles of Knowledge Addition and Knowledge Confirmation (or Belief Enhancement/Withdrawal). However, one problem noted was that these principles are in conflict with each other. It is desirable to be able to use a fusion function to emphasize either Knowledge Addition, or Knowledge Confirmation, or some combination of them, depending upon the requirements of the particular application. Proceeding from these two basic requirements of what a fusion method should accomplish, in this chapter an analytical fusion function will be derived which combines both principles and incorporates flexibility for situations requiring varying ratios of Knowledge

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Addition versus Belief Enhancement/Withdrawal. The ratio of the influence of these two principles in the fusion process is controlled by a single parameter.

6.2 Mathematical Knowledge Addition — Knowledge Confirmation

Conflict Resolution

In this section, the two data sets to be fused are considered to be two images produced by two sensors, $S_1$ and $S_2$. These images are assumed to be registered; i.e., features in one image are located at corresponding locations in the other image. A method of conciliating the two opposing principles of Knowledge Addition and Belief Enhancement/Withdrawal for such images is developed, using the Calculus of Variations [143].

To show how this may be accomplished, let the input image from sensor $S_1$ be expressed by $I_1(x,y)$, where $x$ and $y$ denote the pixel or foveal location in the image. Here, for mathematical development, continuous functions of $x$ and $y$ are assumed. Similarly, let $I_2(x,y)$ denote the input image from sensor $S_2$. These two images are assumed to be correlated to some degree, and registered. Also, they are assumed to both be scaled from zero to one. The fused result will be a function of the two inputs: $Z(x,y) = F(I_1(x,y), I_2(x,y))$. Assuming $x$ and $y$ continuous and $F$ analytic, $Z$ can be approximated to any desired degree by a Taylor series:
\[ Z = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} c_{ij} I_1^i(x,y) I_2^j(x,y) . \]

Here, the \( c_{ij} \) are unknown coefficients which must be determined. \( I_1^i \) and \( I_2^j \) represent the \( i \)th and \( j \)th powers of \( I_1 \) and \( I_2 \), respectively. Since the \( c_{ij} \)'s decrease in magnitude as \( (i + j) \) increases, higher order \( c_{ij} \)'s can be ignored. In order to determine the \( c_{ij} \)'s which are kept, constraints (such as the two previously mentioned) must be imposed. It has been shown [139] that for a second order approximation of \( Z \), more constraints than the two above are required to determine all coefficients. Constraints should obviously not be arbitrarily imposed; they should be meaningful. Rather than attempting to create other constraints and to impose some meaning upon them, the two above meaningful constraints are used to obtain a first order approximation of \( Z \):

\[ Z = c_{00} + c_{10} I_1 + c_{01} I_2 . \]

It is further assumed that the inputs \( I_1 \) and \( I_2 \) have been scaled from zero to one. The coefficient \( c_{00} \) can be ignored since it is not dependent on either input. There are now two unknowns to determine, \( c_{10} \) and \( c_{01} \). The number of unknowns can be reduced to one by normalizing the output so that \( 0 \leq Z \leq 1 \). Since the inputs are scaled from zero to one, this scaling of the output is desirable:

\[
Z = \frac{c_{10} I_1 + c_{01} I_2}{c_{10} + c_{01}} \\
Z = \alpha I_1 + \beta I_2 , \tag{6.1}
\]
where $\alpha = c_{10}/(c_{10} + c_{01})$, $\beta = c_{01}/(c_{10} + c_{01})$ and

$$\alpha + \beta = 1. \tag{6.2}$$

In order to implement the principle of Knowledge Addition, $Z$ should be constrained to be as near to its maximum value, 1, as possible. This is accomplished by minimizing $(1 - Z)$, as below:

$$\min \left[ \int \int_{image} (1 - Z)^2 \, dx \, dy \right]. \tag{6.3}$$

Now, consider the principle of Knowledge Confirmation. If both images contain the same feature, this feature should appear in the output, $Z$. In such a case, the input images would vary at the location of that feature. The difference in the derivatives of the images at this location should be minimal. If only one image contains the feature, the difference in derivatives should be large, and the feature should not appear in the output. For a feature to appear in the output, the difference in derivatives should be minimal. This is expressed as follows:

$$\min \left[ \int \int_{image} \| \nabla (\alpha I_1 - \beta I_2) \|^2 \, dx \, dy \right], \tag{6.4}$$

where $\nabla$ signifies

$$\begin{bmatrix} \partial/\partial x \\ \partial/\partial y \end{bmatrix}.$$

So, the three constraints consist of equations 6.2 – 6.4. In Eq. 6.3, let

$$1 - Z = 1 - (\alpha I_1 + \beta I_2)$$

$$= 1 - (\alpha I_1 + (1 - \alpha)I_2)$$

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\begin{align*}
&= 1 - \alpha I_1 - I_2 + \alpha I_2 \\
&= 1 - I_2 + (I_2 - I_1)\alpha \\
&= f.
\end{align*}

In Eq. 6.4,
\begin{align*}
||\nabla (\alpha I_1 - \beta I_2)||^2 &= (\alpha I_1 - \beta I_2)_x^2 + (\alpha I_1 - \beta I_2)_y^2 \\
&= (\alpha I_1(1 - \alpha)I_2)_x^2 + (\alpha I_1 - (1 - \alpha)I_2)_y^2 \\
&= (\alpha(I_1 + I_2) - I_2)_x^2 + (\alpha(I_1 + I_2) - I_2)_y^2 \\
&= [(\alpha_x(I_1 + I_2) + \alpha(I_{1x} + I_{2x}) - I_{2x})^2 + \\
&[\alpha_y(I_1 + I_2) + \alpha(I_{1y} + I_{2y}) - I_{2y}]^2 \\
&= g_1^2 + g_2^2 ,
\end{align*}

where
\begin{align*}
g_1 &= \alpha_x(I_1 + I_2) + \alpha(I_{1x} + I_{2x}) - I_{2x} \\
g_2 &= \alpha_y(I_1 + I_2) + \alpha(I_{1y} + I_{2y}) - I_{2y} .
\end{align*}

The first constraint, \(\alpha + \beta = 1\), is accomplished by simply writing \(\beta\) in terms of \(\alpha\): \(\beta = 1 - \alpha\). The other constraints can be accomplished by minimizing the integral
\begin{equation}
\int\int_{\text{image}} G \, dx \, dy ,
\end{equation}

where
\begin{equation*}
G = f^2 + \lambda^2(g_1^2 + g_2^2) .
\end{equation*}
The constant $\lambda$ is included to allow the emphasis of either of the two principles (Knowledge Addition or Verification) by varying $\lambda$.

The goal is to find the $\alpha = \alpha(x,y)$ which minimizes Eq. 6.5. This $\alpha$ must satisfy the Euler–Lagrange partial differential equation [143]:

$$\frac{\partial G}{\partial \alpha} - \frac{\partial}{\partial x} \left( \frac{\partial G}{\partial \alpha_x} \right) - \frac{\partial}{\partial y} \left( \frac{\partial G}{\partial \alpha_y} \right) = 0,$$

(6.6)

where

$$G = f^2 + \lambda^2(g_1^2 + g_2^2)$$

$$f = 1 - I_2 - (I_1 - I_2)\alpha$$

$$g_1 = (I_1 + I_2)\alpha_x + (I_{1x} + I_{2x})\alpha - I_{2x}$$

$$g_2 = (I_1 + I_2)\alpha_y + (I_{1y} + I_{2y})\alpha - I_{2y}.$$

So, writing out $G$ explicitly,

$$G = (1 - I_2 - (I_1 - I_2)\alpha)^2 + \lambda^2[((I_1 + I_2)\alpha_x + (I_{1x} + I_{2x})\alpha - I_{2x})^2$$

$$+((I_1 + I_2)\alpha_y + (I_{1y} + I_{2y})\alpha - I_{2y})^2]$$

$$\frac{\partial G}{\partial \alpha} = 2(I_2 - I_1)f + 2\lambda^2[(I_{1x} + I_{2x})g_1 + (I_{1y} + I_{2y})g_2]$$

(6.7)

$$\frac{\partial G}{\partial \alpha_x} = 2\lambda^2(I_1 + I_2)g_1$$

$$\frac{\partial}{\partial x} \left( \frac{\partial G}{\partial \alpha_x} \right) = 2\lambda^2[(I_{1x} + I_{2x})g_1 + (I_1 + I_2)\frac{\partial g_1}{\partial x}$$

(6.8)

$$\frac{\partial G}{\partial \alpha_y} = 2\lambda^2(I_1 + I_2)g_2$$

$$\frac{\partial}{\partial y} \left( \frac{\partial G}{\partial \alpha_y} \right) = 2\lambda^2[(I_{1y} + I_{2y})g_2 + (I_1 + I_2)\frac{\partial g_2}{\partial y}.$$

(6.9)
After substituting these values (Eqs. 6.7 – 6.9) into Eq. 6.5, the result is

\[ 2(I_2 - I_1)f + 2\lambda^2(I_{1x} + I_{2x})g_1 + 2\lambda^2(I_{1y} + I_{2y})g_2 \]

\[ -2\lambda^2(I_{1x} + I_{2x})g_1 + (I_1 + I_2)\frac{\partial g_1}{\partial x} \]

\[ -2\lambda^2(I_{1y} + I_{2y})g_2 + (I_1 + I_2)\frac{\partial g_2}{\partial y} = 0. \]

This equation can be simplified as follows:

\[ 2(I_2 - I_1)f + 2\lambda^2(I_{1x} + I_{2x})g_1 + 2\lambda^2(I_{1y} + I_{2y})g_2 \]

\[ -2\lambda^2(I_{1x} + I_{2x})g_1 - 2\lambda^2(I_1 + I_2)\frac{\partial g_1}{\partial x} \]

\[ -2\lambda^2(I_{1y} + I_{2y})g_2 - 2\lambda^2(I_1 + I_2)\frac{\partial g_2}{\partial y} = 0, \]

or

\[ 2(I_2 - I_1)f - 2\lambda^2(I_1 + I_2)\frac{\partial g_1}{\partial x} - 2\lambda^2(I_1 + I_2)\frac{\partial g_2}{\partial y} = 0 \]

\[ 2(I_2 - I_1)f - 2\lambda^2(I_1 + I_2)(\frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y}) = 0 \]

\[ (I_2 - I_1)f - \lambda^2(I_1 + I_2)(\frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y}) = 0 \]

\[ -(I_2 - I_1)f + \lambda^2(I_1 + I_2)(\frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y}) = 0 \]

\[ (I_1 - I_2)f + \lambda^2(I_1 + I_2)(\frac{\partial g_1}{\partial x} + \frac{\partial g_2}{\partial y}) = 0, \quad (6.10) \]

where

\[ f = 1 - I_2 - (I_1 - I_2)\alpha \]

\[ g_1 = (I_1 + I_2)\alpha_x + (I_{1x} + I_{2x})\alpha - I_{2x} \]
\[ g_2 = (I_1 + I_2)\alpha + (I_{1y} + I_{2y})\alpha - I_{2y} \]
\[
\frac{\partial g_1}{\partial x} = (I_{1x} + I_{2x})\alpha_x + (I_1 + I_2)\alpha_{xx} + (I_{1xx} + I_{2xx})\alpha + \\
+ (I_{1x} + I_{2x})\alpha_x - I_{2xx}
\]
\[
\frac{\partial g_2}{\partial y} = (I_{1y} + I_{2y})\alpha_y + (I_1 + I_2)\alpha_{yy} + (I_{1yy} + I_{2yy})\alpha + \\
+ (I_{1y} + I_{2y})\alpha_y - I_{2yy} .
\]

Now, substituting explicit expressions for \(\partial g_1/\partial x\), \(\partial g_2/\partial y\), and \(f\) into Eq. 6.10, results in

\[
(I_1 - I_2)(1 - I_2 - (I_1 - I_2)\alpha) + \\
\lambda^2(I_1 + I_2)(I_{1x} + I_{2x})\alpha_x + (I_1 + I_2)\alpha_{xx} + (I_{1xx} + I_{2xx})\alpha + \\
(I_{1x} + I_{2x})\alpha_x - I_{2xx} + (I_{1y} + I_{2y})\alpha_y + (I_1 + I_2)\alpha_{yy} + \\
(I_{1yy} + I_{2yy})\alpha + (I_{1y} + I_{2y})\alpha_y - I_{2yy} = 0 .
\]

Simplifying,

\[
I_1 - I_1I_2 - I_1(I_1 - I_2)\alpha - I_2 + I_1^2 + I_2(I_1 - I_2)\alpha + \\
\lambda^2(I_1 + I_2)(I_{1xx} + I_{2xx} + I_{1yy} + I_{2yy})\alpha + \\
\lambda^2(I_1 + I_2)(I_{1x} + 2I_{2x}\alpha_x + \\
\lambda^2(I_1 + I_2)(I_{1y} + 2I_{2y}\alpha_y + \\
\lambda^2(I_1 + I_2)^2\alpha_{xx} + \lambda^2(I_1 + I_2)^2\alpha_{yy} + \\
\lambda^2(I_1 + I_2)(-I_{2xx} - I_{2yy}) = 0 .
\]

Similarly,

\[
[(I_1 - I_1I_2 - I_2 + I_2^2) + \lambda^2(I_1 + I_2)(-I_{2xx} - I_{2yy})] + \\
\]

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\[ \lambda^2 (I_1 + I_2)(I_{1xx} + I_{2xx} + I_{1yy} + I_{2yy}) + \\
I_2(I_1 - I_2) - I_1(I_1 - I_2) \alpha + \\
[2\lambda^2(I_1 + I_2)(I_{1x} + I_{2x})] \alpha_x + \\
[2\lambda^2(I_1 + I_2)(I_{1y} + I_{2y})] \alpha_y + \\
\lambda^2(I_1 + I_2)^2 \alpha_{xx} + \\
\lambda^2(I_1 + I_2)^2 \alpha_{yy} = 0, \]

or

\[
\begin{bmatrix}
(I_1 - I_2)(1 - I_2) + \frac{(-I_{1xx} - I_{2yy})}{\lambda^2(I_1 + I_2)^2} \\
\frac{I_{1xx} + I_{2xx} + I_{1yy} + I_{2yy}}{I_1 + I_2} + \frac{(I_1 - I_2)^2}{\lambda^2(I_1 + I_2)^2} \\
2(I_{1x} + I_{2x}) \\
2(I_{1y} + I_{2y})
\end{bmatrix} \alpha_x + \\
\left( \frac{2(I_{1x} + I_{2x})}{I_1 + I_2} \right) \alpha_y + \alpha_{xx} + \alpha_{yy} = 0.
\]

So,

\[ \alpha_{xx} + \alpha_{yy} + A_1 \alpha_x + A_2 \alpha_y + A_3 \alpha = C. \quad (6.11) \]

Finally,

\[ \nabla^2 \alpha + A_1 \alpha_x + A_2 \alpha_y + A_3 \alpha = C, \quad (6.12) \]

where

\[
A_1 = \frac{2(I_{1x} + I_{2x})}{I_1 + I_2}, \\
A_2 = \frac{2(I_{1y} + I_{2y})}{I_1 + I_2}.
\]

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\[ A_3 = \frac{\nabla^2(I_1 + I_2)}{I_1 + I_2} - \frac{(I_1 - I_2)^2}{\lambda^2(I_1 + I_2)^2} \]
\[ C = \frac{(I_2 - 1)(I_1 - I_2)}{\lambda^2(I_1 + I_2)^2} + \frac{\nabla^2 I_2}{(I_1 + I_2)}. \]

Equation 6.12 must be solved for \( \alpha \), and then fusion performed according to

\[ Z(x, y) = \alpha(x, y)I_1(x, y) + (1 - \alpha(x, y))I_2(x, y). \]
CHAPTER 7

Experimental Results

Contact images can be acquired by means of active touch, in which the sensing robot probes an object. This has been done using a Cincinatti Milacron T³–726 industrial robot equipped with various sensors, including a Lord Corporation F/T Series Model 30/100 force/torque sensor. The object probed was an industrial valve handle. The data gathered were then used in fusion experiments utilizing the 16 methods described in Chapter 5, as well as the analytic method. Results are described below.

7.1 Data Acquisition

The data were gathered in the following manner. First, an industrial robot (Cincinatti Milacron T³–726) equipped with a force/torque sensor (Lord Corporation F/T Series Model 30/100) was programmed to probe an object. A large industrial valve handle was chosen as the object to be probed because it presented well-defined shape characteristics [Fig. 7.1], as well as the fact that it occurs naturally in industrial environments.

For convenience in stabilizing the object, it was positioned with the plane containing its circumference flat on top of a table. The horizontal plane con-
Figure 7.1: Object probed by robot – large valve handle.
taining the table top was divided into a grid consisting of 64 equally spaced rows and 64 columns, the targets for the robot being the points defined by the intersections of rows and columns. A right-handed orthogonal rectangular coordinate system was defined with the $x$- and $y$- directions in this plane, and the positive $z$- axis directed upward.

The robot end-effector, grasping a tool for probing, moved above the object over the first point of the grid. The end-effector, gripping the probe, then moved downward toward the table from directly above while monitoring the forces and torques experienced by the probe. Motion was halted when contact with either the object, or the table, was sensed. The height of the point of contact above the table was stored in an array location corresponding to the particular grid point probed. The forces detected at the time of contact in the $x$-, $y$-, and $z$-directions were likewise stored, as well as the torques about the $x$-, $y$-, and $z$-axes of the end-effector. The robot then raised the probe, moved above the next grid point, and repeated the process for all grid points. Thus, seven pieces of information – three forces, three torques, and one height (range, or depth) – were measured at each point of the two-dimensional grid. This information was then scaled from 0 to 255 and stored as seven images, illustrated in Figs. 7.2 through 7.8.

In these seven images, the positive $x$-axis is oriented vertically in the image plane, while the positive $y$-axis is oriented horizontally to the left. The positive $z$-axis extends upward out of the plane of the image. The depth image is shown
Figure 7.2: Force image constructed by measuring $x$-components of reaction force as robot probes object.
Figure 7.3: Force image constructed by measuring $y$-components of reaction force as robot probes object.
Figure 7.4: Force image constructed by measuring $z$-components of reaction force as robot probes object.
Figure 7.5: Torque image constructed by measuring torque about x-axis as robot probes object.
Figure 7.6: Torque image constructed by measuring torque about y-axis as robot probes object.
Figure 7.7: Torque image constructed by measuring torque about z-axis as robot probes object.
Figure 7.8: *Image obtained by measuring height at which motion of robot’s end-effector probe was halted by contact with object.*
in Fig. 7.8. Figure 7.2 shows the image created by storing the force measured in the $x$–direction. The force measured in the $y$–direction is illustrated in Fig. 7.3, while the image shown in Fig. 7.4 illustrates the $z$–component of the reaction force. Figure 7.5 is the image showing the torque about the $x$–axis, Fig. 7.6 shows the torque about the $y$–axis, and Fig. 7.7 shows the the torque about the $z$–axis.

### 7.2 Experimental Fusion Results

This section is a comparison of the evidential reasoning based methods described in Chapter 5, and the analytic method described in Chapter 6. All these fusion methods were implemented in the C language, and run on a VAX–11/785 computer. Experimental results are examined.

The inputs to the fusion methods are two $64 \times 64$ edge maps. Contact images were obtained as described in the previous section. For this experiment, the images consisting of the force information in the $x$– and $y$–directions ($F_x$ and $F_y$) were used (Figs. 7.2 and 7.3). The Canny edge detector algorithm [105] was applied to those images, resulting in Figs. 7.9 and 7.10. This feature extraction phase is arbitrary because any other operator could have been implemented.

The evidential reasoning based methods listed in Table 5.1 were then used to fuse the two edge maps. The results are shown in Fig. 7.11. One basis for comparing these results is to consider the relative amounts of Knowledge
Figure 7.9: Edge map obtained by applying Canny edge detector to force image constructed by measuring horizontal components of reaction force as robot probes object.
Figure 7.10: Edge map obtained by applying Canny edge detector to force image constructed by measuring vertical components of reaction force as robot probes object.
Figure 7.11: Fusion results obtained by evidential reasoning techniques.
Addition and Knowledge Verification involved in the fusion process. This can be done by considering the ideal volume and actual volume defined in Chapter 5.

Starting at the upper left hand corner of Fig. 7.11, and moving right, row by row, these images correspond to the methods listed in Table 5.2. The gradual change from Belief Enhancement/Withdrawal to Knowledge Addition is apparent since these functions were arranged according to increasing actual volume. The ideal volume also increases, but there are a few discrepancies between the ordering of these two measures. These discrepancies are due to the effect of the inputs. Some fusion functions, for example, may weight a certain range of input values more highly than other ranges. In this case, if the inputs contain primarily low values, the actual volume may be lower than that of another function, while inputs containing higher values may result in a different ordering of actual volumes.

The two input edge maps were also fused using the analytic method derived in Chapter 6 [Fig. 7.12]. Here, a similar pattern exists, in which the results range from Belief Enhancement/Withdrawal to Knowledge Addition, with several intermediate results between them. These results were obtained by varying the parameter $\lambda$ from 1000 to .001. The values of $\lambda$ used, corresponding to the images in Fig. 7.12, are listed in Table 7.1. As previously mentioned, $\lambda$ is a weighting factor, allowing one to emphasize either of the two fundamental principles of Knowledge Addition or of Belief Enhancement/Withdrawal.
Figure 7.12: Fusion results obtained by analytic method described in Chapter 6
Table 7.1: Values of lambda used in analytic fusion with corresponding volumes

<table>
<thead>
<tr>
<th>Lambda</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>100.0</td>
<td>0.049</td>
</tr>
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<td>0.050</td>
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<tr>
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<tr>
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</tbody>
</table>
CHAPTER 8

Summary and Conclusions

8.1 Summary

In this thesis, the need for robotics and sensing, in general, and for robotic multi-sensory fusion, in particular, has been examined. It has been seen that robots can protect humans from hazardous environments, among other useful functions. It has been shown that sensing in robotics can provide flexibility and adaptability in the use of robots. Contact sensing was seen to be of great utility, especially in robotic manipulation, an extremely common robotic function. It was seen that often, robots can perform their tasks more efficiently and accurately when provided with information from several different types of sensors. This use of multiple sensors, however, requires the use of some sensor fusion methodology in order to effectively utilize the information they provide. It was also shown that sensor fusion, although not needed in every application, is useful in many.

Since contact data were acquired and used in the fusion experiments described in this thesis, the field of sensing was narrowed to contact sensing, which was examined in some detail. Different types of transducers were de-
scribed, including piezoelectric, magnetoelastic, optical, ultrasonic pulse–echo, and resistive strain gauges. In many of these sensors, force causes some type of motion, which results in a change of resistance, voltage, capacitance, magnetic reluctance, or some other measurable property.

Several criteria for the design of contact sensors were examined, including ruggedness, resolution, low hysteresis, etc. Since contact sensors constitute the interface between a robot and the environment in which it is to be used, a major consideration in the choice or design of a particular sensor should be the nature of this environment.

The sensor chosen to gather the data for the fusion experiments described in this thesis was a wrist–mounted force/torque sensor employing strain gauges. It is capable of sensing contact and of measuring six components of force and torque. This capability allowed the construction of six force/torque images, in addition to the depth image created by the end–effector position data (depth information) provided by the robot itself.

An important consideration in these experiments was the sensing strategy used in acquiring the contact data. Passive and active sensing methods were described. Active contact sensing, involving motion of the robot, was chosen. Some researchers have attempted to improve upon simple active sensing, in which the robot's moves are pre–programmed, by the use of Intelligent Machine Perception, in which the robot chooses its own moves upon the basis of previously–acquired information. Several researchers' methods of Intelligent Ma-
chine Perception were described. This is useful when relatively little is known about the size, shape, orientation, or location of a target object, and precise moves cannot be specified. However, when more information is known about the object's size, shape, orientation, and location, simple active sensing will suffice. Simple active sensing also requires less computation, and thus, all other things being equal, should be faster. Since the goal in these experiments was to probe the entire surface of an object, whose general shape, size, and location were known, Intelligent Machine Perception offered no great advantages, and simple active sensing was used.

Methods for processing contact data obtained by some sensing strategy, and for extracting information from it were examined. It was seen that, particularly in the case of tactile images, methods commonly used with vision images were also suitable for use with contact data. Thus, an edge detector algorithm [105], was used to extract edges from the contact data gathered for the experiments described in this thesis.

Desirable characteristics of sensor fusion functions were investigated. In several robotic sensor fusion methods, complementary sensors are utilized, in which one sensor provides information not provided by others, thus illustrating the principle of Knowledge Addition, one of two basic principles typically desired in the fusion of data. The other principle, that of Knowledge Confirmation, is of value in reducing noise or verifying questionable information. An analytic fusion method was derived to incorporate any desired ratio of these two princi-
cies. Problems with some of these methods were noted, and experiments were conducted with real data to compare several of these methods.

It was shown that different evidential reasoning based fusion functions favored each of the two aforementioned principles to varying degrees, depending upon the particular fusion function employed. The analytic method also favored each of the two principles to varying degrees, depending upon the value of the parameter $\lambda$ selected.

8.2 Conclusions

In comparing the results of the 16 evidential reasoning based methods with those of the analytic method, results from the 16 evidential reasoning based methods were ordered according to their position between the two extremes of Knowledge Addition and Knowledge Confirmation. Results from the analytic method were similarly ordered. It can be seen that a major difference between the 16 evidential reasoning based functions was in the relative amount of Knowledge Addition versus Knowledge Confirmation each employed. The analytic method allows the explicit specification of any desired amount of Knowledge Addition versus Knowledge Confirmation.

The analytic method covers the entire range from total Knowledge Addition to total Knowledge Confirmation in a continuous manner. Any desired "mix" of these two extremes is easily attainable. The principle of Knowledge Addi-
tion tends to increase, never to decrease, the taxel value, while the principle of Knowledge Confirmation tends to decrease the taxel value, never to increase it. Therefore, fusion performed utilizing the principle of Knowledge Addition should result in an output having higher values than the output of fusion performed utilizing the other principle. The more Knowledge Addition is favored, the higher the output values, and thus, the greater the volume underneath the two-dimensional output surface. Of course, the output is not only dependent upon the fusion methodology, but also upon the inputs. In order to facilitate the comparison of various fusion methods, another volume, the *ideal volume*, which is independent of the inputs, was used. As expected, the more Knowledge Addition is favored, the larger this volume [Table 5.2]. The volumes under the actual fused results for the evidential reasoning based methods are also shown in Table 5.2, and vary from high to low, corresponding to the amount of Knowledge Addition seen in the outputs.

There are several ways of normalizing these volumes, or making them independent of the inputs. The concept of *ideal volume* was used for this purpose in this thesis. Another one is given below:

\[
\text{volume} = \frac{\left[ \int \int_{\text{image}} f(x,y) \, dx \, dy \right]^2}{\left[ \int \int_{\text{image}} i_1(x,y) \, dx \, dy \right] \left[ \int \int_{\text{image}} i_2(x,y) \, dx \, dy \right]}
\]

The method used in this thesis to normalize volume, that of the *ideal volume* described in Chapter 5, is computationally simpler than the above. It also yields
results which differ by only a constant factor from results obtained by the above measure. Even the application of the above measure to the actual volumes would alter the actual volumes by only a constant factor, since the same inputs were used for all fusion functions. Therefore, this measure was not used.

If it is desired to utilize a certain ratio of Knowledge Addition versus Knowledge Confirmation in fusion, one approach towards attaining this goal would be to examine existing fusion functions in a search for the one yielding the desired ratio. In practice, this ratio would most likely not be attainable, only an approximation to it. If such an approximation were not suitable, another approach would be the derivation of a new fusion function with the desired ratio. This second approach, however, may not be easily attained, or may even be impossible. Even if the desired ratio were attained, the new fusion method might have additional undesired characteristics, such as uneven weighting of different ranges of input values, as can be seen in several of the 16 fusion functions illustrated in Chapter 5. The analytic method avoids these difficulties by allowing the user to obtain any desired degree of Knowledge Addition versus Knowledge Confirmation by the use of only one function. Merely varying the single parameter \( \lambda \), instead of replacing an entire fusion function with another function, allows control of all factors while varying only the one of interest. It should be noted that \( \lambda \) is continuous, and one can theoretically choose from an infinite number of values. The only limitation to this in practice is the limit imposed by the number of significant figures allowed by the particular computing equipment.
used to implement the method. Thus, if in practice, which of the two principles needs to be emphasized is known, the analytic method seems appropriate.

On the other hand, if the degree of balance desired between the two principles is not known, trial-and-error may be employed to allow the user to consider previous outputs to determine whether more Knowledge Confirmation, or less, is called for. Several different ratios of these two principles might be experimented with, basing each successive ratio upon the results obtained using previous ratios. Here, the user needs a convenient means of altering the balance between these two principles in a specified direction and by varying amounts. The variation of the single parameter in the analytic method provides such specific control over the direction and amount of change in the balance between these two conflicting principles. In this case, it is obviously more convenient to vary one parameter than to continuously search for and/or derive appropriate fusion functions. Given only slight experience with the analytic method, previous results can give a user not only the desired direction of change in \( \lambda \), but also an estimate of the desired magnitude of change. However, when using other methods than the analytic one in the process of successively varying the ratio of the two fusion principles, such guidance from previous results does not exist. The user may know the desired direction of change in the ratio of the principles, but this information is not directly and easily utilized as it is with the analytic method.

Mathematically, the analytic method is unique in that it follows directly from
the two basic principles of data fusion and the need in practice to simultaneously employ both. These two principles are both fundamental and explicit in, both its derivation and its application. It is important to note that undesired extraneous influences are not present in the derivation; these two principles are not only the basis of the method, but the only basis of the method. These two basic principles, however, are not explicit in the formulation of the evidential reasoning based functions. For these functions, there is no convenient way to shift the emphasis between the two basic principles without the use of another function. The analytic method, however, appears not only mathematically elegant, but very versatile, allowing the selection of an infinite number of degrees of Knowledge Addition versus Knowledge Confirmation by simply varying a single parameter.

Not only did the evidential reasoning based methods suffer from the disadvantage of lack of ease in choosing the degree of conformity to the two principles mentioned above, but these functions also suffered from several other problems: lack of continuity, nondifferentiability, etc. The analytic method, however, is based upon the assumption that the output is analytic, and therefore, both continuous and differentiable.

The analytic fusion method is seen to possess several advantages: (1) It allows a large (theoretically infinite) choice of values for the ratio of the two basic principles involved in data fusion. (2) It allows the user to specify exactly this ratio, not only approximately, as when other methods are used. (3) This ratio may be singled out and varied without also inadvertently varying other
factors, as may occur with other methods. (4) Another great benefit of the method is the ease, convenience, and savings of time which it allows in "trial-and-error" searching for the most appropriate fusion method for a particular application. One need only vary a single parameter, instead of searching for (or deriving) an entirely different function. (5) The output is "smooth;" both continuous and differentiable. (6) The method can be used with a variety of types of data. (Perhaps the most obvious type is vision or intensity, but that other types may also be used is demonstrated by the use of force data in this thesis.)

It has been shown that multi-sensor fusion is of great value in robotics. The choice of an appropriate fusion methodology is therefore of utmost importance. Fusion can serve to accomplish either of two contradictory purposes: Knowledge Addition or Knowledge Confirmation. In fact, most existing fusion methods lie somewhere between these two principles, which are opposite extremes of a continuum. Any other effects which may be desired in a fusion function, such as the emphasis of certain image areas, or features, or of one input over another, may best be obtained by simply weighting or preprocessing the inputs (e.g., the edge extraction performed in this work). These two principles are fundamental to the fusion process itself. Therefore, a fusion methodology incorporating both of these principles and allowing any desired ratio of them is highly desirable. It is possible to derive such a fusion method which allows the specification of any desired ratio of both of these principles. Such a function was derived, implemented,
and shown to perform as predicted on real data. The results, both figures and numerical data, illustrate the varying ratios of the two principles, corresponding to the value of the weighting parameter used. The analytic method provides the only means of attaining such accuracy in specifying the exact ratio of the two basic fusion principles. Since the ratio of the two fusion principles desired is the major characteristic of fusion functions, it is certainly more desirable that the choice of this characteristic be made conveniently, simply, with great speed and convenience, as well as with the ability to specify it with as much accuracy as possible, than that it be made by “trial-and-error,” with no guarantee of finding or deriving the exactly specified desired qualities, but only the hope of coming close to them, and possibly other problems such as the inclusion of undesired extraneous factors due to the use of a different function.

Therefore, the analytic fusion function is seen to be a method showing great promise and utility for the future in providing convenience and greater accuracy in specification of the desired ratio of basic fusion principles than previously possible. Such a fusion methodology has a wide range of potential applications in sensor/data fusion, some of which are discussed in the next section.

One possible drawback to the analytic method, when compared with the other methods considered, is the time involved in initially implementing the method. The 16 evidential reasoning based functions were relatively simple and easy to implement. However, once the analytic method is implemented, its versatility and flexibility should justify the effort. Then, a variety of different
fusion functions can be obtained very easily, by simply varying the parameter \( \lambda \).

8.3 Future Work

It may be desired at times to utilize different ratios of the two basic fusion principles in different areas of the same image. For example, when fusing range and vision data, and it is known that in a certain area of the image a written message should appear, such as a package label, the principle of Knowledge Addition should be favored more in that area of the image than in other areas. Future research should consider defining a separate \( \lambda \) for different areas of the input images, or even for each taxel. This would impose more computational burden. However, parallel implementations of this method are possible, and constitute another area for research. The analytic method described in this thesis has been successfully implemented on a parallel processing system [140].

The criterion for similarity of the two inputs for Knowledge Confirmation was implemented by minimizing the difference in derivatives. Other criteria for similarity, such as a minimum difference in magnitude of the two inputs, or some weighted combination of several criteria, could be explored in future research. Future work might also consider the evaluation of the analytic method with respect to other characteristics mentioned in Chapter 4.

Potential applications for the analytic fusion method constitute another area of research. The analytic fusion method is potentially applicable in a wide
range of applications. Remote sensing is one such area. Satellite images of
the earth may be fused with other images of the same type and taken at the
same time in order to eliminate noise by emphasizing the principle of Knowledge
Confirmation. Images taken at different times of the year, but of the same area,
may also be fused utilizing this same principle in order to separate constant
features from transient ones. Also, sometimes it is desired to locate objects
which vary in the wavelengths of radiation they reflect and/or emit. The use
of multispectral images emphasizing Knowledge Addition is appropriate for this
application. Robot navigation is another area of potential application. Mobile
robots need to verify the existence of target objects, doors, etc., as well as to
gather additional information from more than one sensor about different regions
of their surroundings. Thus, mobile robots need to utilize both principles. This
method is applicable with image data of varying types and combinations: vision,
force (used in this thesis), torque, range, infra-red, tactile, etc.

Not only is it applicable with various image types, but also with non–image
data. For example, in the field of avionics instrumentation, certain instruments
may give warning signals indicating certain actions should not be undertaken.
If it is desirable that no action be taken if even a single warning is present,
then these warning signals could be combined using the principle of Knowledge
Addition. On the other hand, if certain “ready” signals are desired before certain
actions can be taken, these signals can be combined utilizing the principle of
Knowledge Verification. A simple logical AND or logical OR could be used to
implement these functions, but the flexibility of the analytic method allows for the design of various systems allowing different degrees of “warning” signals and of “ready” signals. This could be very useful in quickly summarizing information from many instruments into a single continuous (not binary) reading. This would have the double benefit of giving the pilot more information than a simple binary “ready” or “warning” signal, while saving him the valuable time required to read numerous instruments. The versatility of this approach has already been illustrated by its application to the fusion of contact data (described in this thesis), vision and range data [136], and color images [138].
BIBLIOGRAPHY
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VITA

Joel Wythe Spears is a native of East Tennessee, born in Kingsport, Tennessee, where he attended Ketron High School and graduated 15th in his class, majoring in math and science. He also participated in the school band and was a member of the Beta Club.

He obtained the B.S. degree in Math at East Tennessee State University in 1982, and a B.S. degree in Electrical and Computer Engineering at the University of Tennessee, Knoxville, in 1986.

He joined the Electrical and Computer Engineering Department at the University of Tennessee, Knoxville, in 1987 as a graduate student. He worked as a research assistant to Dr. M. A. Abidi, and also as a graduate teaching assistant in the Department.

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He is a member of IEEE and Eta Kappa Nu.